

Statement: All numbers are $n = 2^l, l \in \mathbb{N}$.

I. Let us prove that for all even natural numbers. We are claiming that after a certain finite number of steps all elements of the sequence will become even. For us to be able to do that let us form a table of differences of sequence $(a_i)_{i \in \mathbb{N}}$, where $a_i = x_{i+1}$, for $i = 0, 1, \dots, n-1$ and $a_i := a_{i-n}, i \geq n$. It is clear that all sequences of the differences will periodically repeat. Number $a_i^k, 0 \leq i \leq n-1$ is generally equal to $(i+1)$ -member of the sequence obtained after k transformations of the sequence $x_1, x_2, \dots, x_{n-1}, x_n$ (because of the absolute values in the problem), but it is easily seen that those two numbers are of same parity.

Let calculate now the remainder when dividing $a_i^{(2^l)}$ by 2. We have:

$$a_i^{(2^l)} = \sum_{j=0}^{2^l} (-1)^{2^l-j} \binom{2^l}{j} a_{i+j} \equiv a_i + a_{i+2^l} = 2a_i \equiv 0 \pmod{2}.$$

Hence, $a_i^{(2^l)}$ is even for all i and therefore all elements of the sequence obtained from $x_1, x_2, \dots, x_{n-1}, x_n$ with $2l$ operations are even likewise. We can divide them by 2 in our minds and proceed. We will again at some point get only even numbers, what means that all newly obtained elements of the sequence are divisible by 4. If we continue this iteration, we conclude that after a finite number of steps all elements of the sequence will be divisible with an arbitrary exponent of number 2. On the other hand, all elements if the sequence obtained from $x_1, x_2, \dots, x_{n-1}, x_n$ are non-negative and non greater than $\max\{x_1^{(l)}, x_2^{(l)}, \dots, x_n^{(l)}\}$, where $x_1^{(l)}, x_2^{(l)}, \dots, x_n^{(l)}$ are gotten after the first transformation. This is obvious since $x_j^{(l)}, x_{j+1}^{(l)} \geq 0$ and $|x_{j+1}^{(l)} - x_j^{(l)}| \leq \max\{x_j^{(l)}, x_{j+1}^{(l)}\}$

Therefore, we will get to the sequence of all zeroes.

II. Let us prove that the claim is not true for all others natural numbers. Let n be from sequence $x_1, x_2, \dots, x_{n-1}, x_n$ such that we can get only zeroes and not all x_j equal.

Therefore,

$$n = \sum_{j=1}^n \frac{|x'_{j+1} - x'_j|}{a} \equiv \sum_{j=1}^n \frac{x'_{j+1} - x'_j}{a} = 0(\text{mod}2).$$

Hence, n must be even.

Q.E.D.