5200 fall 2009 Vector methods

One technique we did not have time for, that works well in Cartesian space is the algebra of vectors. We take advantage of the presence of a special point, the origin O, and we think of every other point A, as an arrow pointing from O to A. If we have two points A,B, we think of B-A as an arrow pointing from A to B. Then we can add arrows by putting the tail of the second at the head of the first, and then the sum goes from the tail of the first to the head of the second. We can also multiply arrows by numbers by stretching the arrow according to the number, so 2 times an arrow is an arrow twicw as long but in the same direction.

This makes it easy and computational to show that all medians of a triangle meet at a point which is 2/3 of the way up each median from the corresponding vertex, like this. To get from B to C we would add (C-B) to B. To get from B to the midpoint of the segment BC we would add (1/2)(C-B) to B. Thus to get from A to that midpoint we would first add (B-A) to A, getting to B, then add also (1/2)(C-B) to that. So the arrow going from A to the midpoint of segment BC is V = (B-A) + (1/2)(C-B) = (1/2)B + (1/2)C - A. Thus if we add this to A, we get the midpoint (1/2)B + (1/2)C of BC. Now if we only want to get 2/3 of the way from A to that midpoint, we would add only (2/3) of V to A, so we would add (2/3)V = (1/3)B + (1/3)C – (2/3)A to A, getting (1/3)A + (1/3)B + (1/3)C. Now it is easy to believe, from the symmetry of this result, that going (2/3) of the way from any vertex to the oppposite midpoint will giove the same result. I.e. the point where all medians meet is (1/3)A + (1/3)B + (1/3)C, which may also be reached by adding the arrows from the porigin to each vertex as described above, and shrinking the length by (1/3). The resulting arrow will go from the origin to the centroid.

This shows the quite beautiful fact that in coordinates, the midpoint of AB has coordinates (1/2)(A+B), e.g. the midpoint of the segment joining (3,6) and (1,4) is (2,5). Similarly the centroid of triangle ABC has coordinates (1/3)(A+B+C). This also suggests the result we discovered by “Zen” earlier, that the coordinates of the centroid of a tetrahedron (4 sided pyramid) ABCD, are (1/4)(A+B+C+D). There is also no mystery to locating in 4 space the cordinates of the centroid of a pyramid with 5 vertices.