Unit Plan – Lesson #1

- **Topic:** Ellipses
- **Grade Level:** 11 (Math 3)
- **GPS Standards:**
    - **Content Standards:**

      MM3G2. Students will recognize, analyze, and graph the equations of the conic sections (parabolas, circles, ellipses, and hyperbolas).
      b. Graph conic sections, identifying fundamental characteristics.

    - **Process Standards:**

      MM3P3. Students will communicate mathematically.
      a. Organize and consolidate their mathematical thinking through communication.
      b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Mathematical Goals: At the end of this lesson students will be able to construct an ellipse and understand its graphical representation. They will also be able to understand an ellipse as an extension of a circle. Understanding of the graphical representation and the extension from a circle to an ellipse will be evaluated by analyzing the answers students provide to the questions in each activity.

Materials to be used:

- Piece of String tied in a loop
- Two thumbtacks
- Corkboard
- Large Sheet of Paper
- Pencil
- 16 Precut Circles for Folding Exercise
- Two computer desktops with Geometer's Sketchpad (GSP) installed

Prior Knowledge Needed:

- Basic Proficiency in Navigating Geometer's Sketchpad (including use of menus and the tools palette)
- Properties of Segments (including definition, midpoint, perpendicular segments)
- Properties of Circles (including definition, radius, center)

This task builds upon students' previous exposure to and experience with constructing circles. Whereas the circle has one focal point called the center, the ellipse has two "centers" or focal points appropriately called foci. This is the basis for the extension of the concept of circles into the understanding of ellipses. This task will build on students' prior exposure to topics covered in science classes such as planetary motion and pendulum oscillation. These connections are made in the closing discussion.

Potential Challenges to Struggling Students or IEP/ELL Students

Although much consideration has taken place to provide a thorough yet efficient lesson, there are potential challenges that might be met by students who struggle with basic concepts or by students who are either designated as English Language Learners (ELL) or have other Special Education needs. The primary challenge that will be met will be in the usage of mathematical terminology and vocabulary to describe the ideas and concepts covered in the lesson. This issue can be addressed by restating mathematical terminology and vocabulary with simpler language. For example, if a student has difficulty understanding the meaning of the term "foci", then alternate terminology, such as focus points or middle points, can be utilized. When alternate terminology is employed along with drawings and other representations, this challenge can be minimized.

There are also a multitude of potential challenges that can face students with Specific

Learning Disabilities (LD). The best method to pursue to address challenges to these students begins with the teachers in the Special Education program. Collaboration with these professionals will lead to strategies to address specific student concerns in the lesson.

Classroom Setup:

The classroom period is 50 minutes. After an introduction that provides the definition of an ellipse and connections to other academic subjects and everyday life, the students will be split into four groups that will rotate through four stations. Each rotation will last one class period. One station will have the string, thumbtacks, corkboard, large paper, and pencil for manual construction. The second station will have precut paper circles for ellipse construction via a folding exercise. The remaining two stations will each consist of a desktop computer with Geometer's Sketchpad installed for dynamic constructions. Since there are sixteen students in each class, each group will have four students and each station will have four chairs. Each student will be engaged in one of four roles utilized at each station: instruction reader, instruction follower, question reader, and question answerer (writes answers to questions on supplied worksheets). Students will switch roles as they rotate from station to station and will fill each role once.

Lesson Structure

I. Introduction

To introduce the lesson, properties of a circle are reviewed, focusing on the center and reintroducing it as a focus point from which the entire circle is created. The following question is then posed: "What happens to a circle when the focus point (center) of the circle splits into two points?" Students are given an opportunity to make conjectures that are written on the board. The students are instructed to keep these ideas in mind as they progress through the lesson tasks. The tasks will be introduced along with the steps required to complete the tasks. Each group will be handed a packet of four pages, each page having the steps and questions to be answered at the end of each task.

The review of the basic properties of a circle ensures access to all students, since the students, by the time they have reached Math III, have had multiple exposures to circles, circle construction, and properties of circles in middle school math as well as in Math II. Cognitive demand is maintained throughout the task by focusing students' attention on procedures that are utilized to develop a better understanding of ellipses. By following the step-by-step procedures and by answering the questions that follow them, connections are established between students' prior knowledge and the underlying concepts involved with understanding ellipses. The tasks result in graphical representations of ellipses that, in later discussions, will lead to algebraic representations with greater awareness of the connection between the two. The questions that follow the procedures force students to engage with the ideas and concepts so that the task can be completed successfully and understanding is created.

II. Investigation

During the investigation, each group goes to the assigned starting station. The students in the group are allowed to pick the role they will assume in their starting station, with the understanding that each group member must fill a different role at each station until each member has filled all four roles. This change of roles will help to keep students engaged and minimize frustration with the task. Whenever possible, the teacher will have extra steps to follow to extend the activity for students who finish the activity quickly. During the investigation the teacher will monitor group work, only joining the groups and throwing in questions when needed, as well as making notes to select student responses for whole class discussion. Since the students are following "*procedures with connections*" tasks, steps are followed during the investigation phase that lead to solution strategies which are shared and discussed during the Whole Class Discussion Phase.

The tasks for each station are as follows:

<u>Station 1</u> takes the string, the two thumbtacks, the corkboard, paper, and pencil. The paper is placed on the corkboard and the thumbtacks are stuck into it. The string is then tied to each thumbtack, making sure that the string has some slack in it. Each thumbtack will serve as the focal points of the ellipse. The loose string is then pulled taut with the pencil, with the string lying near the tip of the pencil. With the string taut, the pencil is swung around the focal points letting the tip of the pencil trace its path. The pencil is then repositioned to draw the remainder of the curve.

The following questions are asked at Station 1:

- What symmetries are observed in the ellipse? Draw a picture to illustrate any lines of symmetry.
- Move the thumb tacks <u>farther apart</u>. Redraw the ellipse and compare the shape to your original ellipse.
- Move the thumb tacks <u>closer together</u>. Redraw the ellipse and compare the shape to your original ellipse.
- If you move the thumb tacks far apart and make the string taut, what shape will your pencil draw?
- If you attach both ends of the string to one thumbtack, what shape will your pencil draw?

<u>Possible Solution Strategies for Station 1</u>

Students will probably easily see the symmetry formed by the major axis. It may be more difficult to see the symmetry formed by the minor axis. The other questions leave little room for multiple solutions or strategies since the task consists of step-by-step instructions.

Station 2 uses a desktop workstation with Geometer's Sketchpad (GSP) installed on it. A new GSP sketch is opened and a horizontal line is drawn at the top of the screen. Points A, B, and C are constructed on the line with point C located between A and B. The line is hidden and two segments are constructed, AC and CB. Below the two segments, two points, F1 and F2, are created to represent to foci of an ellipse. Two circles are constructed, one using F1 as the center and AC as the length of the radius, the second circle using F2 as the center and CB as the length of the radius. The intersections of both circles are created and traced using the Trace Intersections function in the Display menu. Point C is dragged back and forth between points A and B and, as a result the intersection points move leaving a traced path that is observed as an ellipse. This ellipse construction is repeated after changing the distance between the focal points F1 and F2 and erasing the previous traces. Several new ellipses are traced, each time changing the distance between the focal points. Finally, the Locus function is used to automatically adjust the traces as F1 and F2 are moved.

The following questions are asked at Station 2:

- When point C is dragged between points A and B, the radii of both circles change, but there is still a relationship between the two radii. What is that relationship?
- Why do the two intersection points trace an ellipse?
- How far apart can the focal points be before they no longer trace an ellipse?

Possible Solution Strategies for Station 2

Most students will probably see the relationship between the radii by looking at the segments used to create the circles. Since the segment is the same length, the radii have to add up to the same constant number represented by the segment AB.

The intersections trace an ellipse because as C moves from A to B the distance from A to C becomes larger and the distance from B to C becomes smaller. This creates the curve that is the ellipse.

By moving the circles around and tracing the intersections (or alternatively using the Locus tool), it can be observed that the circles must intersect to create an ellipse. So to trace an ellipse, the foci cannot be moved any farther apart than the sum of the radii.

<u>Station 3</u> utilizes the precut circles to construct an ellipse. The instruction follower, the question reader, and the question answerer at the station will each take one of the precut circles and mark the center as point A. Another random point will be placed inside the circle and be called point B. The circle is folded so that the edge of the circle lands on point B. A sharp crease is created to mark the fold and the circle is unfolded. Several different folds and creases are created in the same way, where the edge of the circle lands directly on point B. After 12 or more creases have been made, they are examined to see if any pattern can be observed. After making several more folds, the creases are observed and discussed within the group, comparing each of the three folded circles.

The following questions are asked at Station 3:

- What do the creases appear to form? What do points A and B represent?
- If you move point B closer to the edge of the circle and fold another curve, how would the shape compare to your original curve?

- If you move point B closer to the center of the circle and fold another curve, how would the shape compare to your original curve?

Possible Solution Strategies for Station 3

If students are folding the circle correctly, then the creases should form an ellipse around the center point A and the random point B. A and B represent the foci of the ellipse formed by the creases.

This task requires at least two circles being folded so that the results can be seen from the alternate placement of the random point B. By comparing two or more circles, students can see that placing point B closer to the center results in an ellipse that is rounder, and placing point B closer to the edge of the circle results in an ellipse that is thinner.

Station 4 uses a desktop workstation with Geometer's Sketchpad (GSP) installed on it to model the circle folding exercise performed at Station 3. A new blank sketch is opened and a large circle is drawn using the Compass tool in the Tools Palette. The center of the circle is labeled as point A. A point B is constructed on the interior of the circle and point C is constructed on the circle's circumference using the Point Tool on the Tools Palette. A segment is constructed between points B and C and a perpendicular bisector is created by constructing a midpoint and then a perpendicular line. This perpendicular line serves as the "crease" created in the circle folding exercise. As point C is dragged along the circle's circumference, the "crease" changes its location. The movement of the crease line can be traced by selecting the line and selecting Trace Line from the Display menu. As point C is dragged around the circle, a collection of crease lines is created. Point B can be repositioned and a new collection of crease lines created. To have GSP automatically adjust the crease lines as B is changed, select point C and the crease line and use the Locus tool. Now B can be moved and the collection of crease lines automatically adjusts.

The following questions are asked at Station 4:

- How does the shape of the curve change as you move point B closer to the edge of the circle?
- How does the shape of the curve change as you move point B closer to the center of the circle?
- Select point B and the circle, then select Edit/Merge to merge point B to the circle. What happens to the crease pattern? Does this remind you anything about your study of circles in Math II?
- Select Edit/Split to split point B from the circle. Now merge point B with the center of the circle. What happens to the crease pattern?

Possible Solution Strategies for Station 4

Since the activity at Station 4 is similar to the activity at Station 3, the results should be similar. The difference between the two activities lies in the tool being used to show a dynamic representation of the circle folding activity. As the random point B is moved inside the circle, the crease marks automatically adjust to reflect the changes resulting from relocating B.

III. Whole Class Discussion of Investigation

Once all four stations have been visited by all four groups, the students will return to their own assigned seats in the class for a discussion of the findings. A representative from one of the four groups (the group is preselected during teacher's monitoring of the activities) will come to the overhead (ELMO document camera) and demonstrate the group findings. The teacher will ask the selected group for the reasoning behind their findings. The teacher will also ask the other groups if their findings were similar or the same and will ask them to explain the reasoning behind different outcomes. This process will repeat, beginning with Station 1 and ending with Station 4.

After the findings from each activity are discussed, questions are posed to make sense of the findings and connect them to the properties of ellipses, expand on the ideas suggested from the findings, and begin to form generalizations from the patterns that are observed.

Questions posed from the <u>Station 1 Activity</u> are:

- When moving the thumbtacks closer together and further apart from each other, two extremes were observed. What shape did you trace when the thumbtacks were moved apart and the string was taut? Why did your trace resemble a line?
- When both ends of the string were tied to the same thumbtack, what shape did you trace? Why did your shape resemble a circle?

Questions posed from the <u>Station 2 Activity</u> are:

- When point C was dragged between points A and B, a relationship between the radii of the two circles was observed. What was that relationship? How did you determine that relationship?

Questions posed from the <u>Station 3 and 4 Activities</u> are:

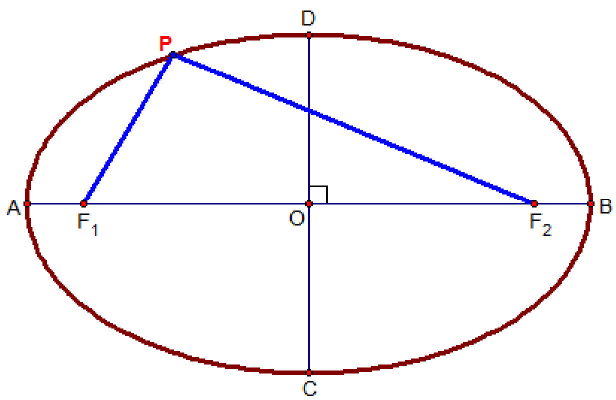
- When point B was merged with the circle, what similar result was observed from the

Station 1 Activity using thumbtacks and string? What caused these results?

- When point B was merged with the center, what similar result was observed from the Station 1 Activity? What caused these results?
- Did anyone move point B outside of the circle? What happened to the crease/Locus pattern? What shape did it form?

IV. Closure and Extension

To wrap up discussion and close the lesson, the key properties of an ellipse are summarized through analysis of conjectures posed during the introduction, applications of the ellipse are discussed, including pendulum oscillation and planetary motion, and an assessment is used which is based on the ellipse construction from Station 1. We use the Promethian Board and open a premade GSP sketch that shows an ellipse with the major axis, minor axis, and focal points.



Distance between F_1 and F_2 = 8 cm.
Length of blue string = 10 cm.

Point P represents the pencil point that is pulling a taut string (represented by segments PF_1 and PF_2) . In this example, the length of the string is 10cm and the distance between the focal points is 8cm.

The following questions are asked as a Ticket out of the Door:

- What is the length of the major axis? Where did you position point P to determine this?
- What is the length of the minor axis? Where did you position point P to determine this?

References

Scher, D. (2002). *Exploring conic sections with the geometer's sketchpad*. (pp. 3-13). Emeryville, CA: Key Curriculum Press.

Conics & sketchpad . (2011, February 15). Retrieved from http://www.youtube.com/watch?v=X3HhpS3m0Cw