Unit Plan – Lesson #2

- **Topic:** Parabolas
- **Grade Level:** 11 (Math 3)
- **GPS Standards:**
  - **Content Standards:**

    MM3G2. Students will recognize, analyze, and graph the equations of the conic sections (parabolas, circles, ellipses, and hyperbolas).
    b. Graph conic sections, identifying fundamental characteristics.

  - **Process Standards:**

    MM3P3. Students will communicate mathematically.
    a. Organize and consolidate their mathematical thinking through communication.
    b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Mathematical Goals: At the end of this lesson students will be able to construct a parabola and understand its graphical representation. They will also be able to understand a parabola from its definition: as the locus of all points equally distant from a point and a line. Understanding of the graphical representation and the definition of a parabola will be evaluated by analyzing the answers students provide to the questions in each activity.

Materials to be used:

- Sixteen pencils
- Sixteen sheets of regular 8 ½ x 11" paper
- Sixteen sheets of paper with concentric circles printed on them
- Two computer desktops with Geometer's Sketchpad (GSP) installed

Prior Knowledge Needed:

- Basic Proficiency in Navigating Geometer's Sketchpad (including use of menus and the tools palette)

- Properties of Segments (including definition, midpoint, perpendicular segments)
- Properties of Lines (including definition, distance to a line)
- Triangle Congruence Postulates (SAS)

Students have been previously exposed to the parabola simply as the resulting graphical representation of a quadratic equation, but have not looked at it from its geometric definition . This task builds upon students' previous exposure to and experience with constructing points and lines in the coordinate plane. Although students have previously worked with these two objects in the coordinate plane separately, they have not yet used them to find other points on the plane. The definition of a parabola as the locus of all points equally distant from a point and a line is the basis for the development of the understanding of parabolas. This task will build on students' prior exposure to parabolas in other academic subjects and in everyday life. These connections are made in the closing discussion.

Potential Challenges to Struggling Students or IEP/ELL Students

Although much consideration has taken place to provide a thorough yet efficient lesson, there are potential challenges that might be met by students who struggle with basic concepts or by students who are either designated as English Language Learners (ELL) or have other Special Education needs. The primary challenge that will be met will be in the usage of mathematical terminology and vocabulary to describe the ideas and concepts covered in the lesson. This issue can be addressed by restating mathematical terminology and vocabulary with simpler language. For example, if a student has difficulty understanding the meaning of the term "equidistant", then alternate terminology, such as "equal distance" or "same distance", can be utilized. When alternate terminology is employed along with drawings and other representations, this challenge can be minimized.

There are also a multitude of potential challenges that can face students with Specific Learning Disabilities (LD). The best method to pursue to address challenges to these students begins with the teachers in the Special Education program. Collaboration with these professionals will lead to strategies to address specific student concerns in the lesson.

Classroom Setup:

The classroom period is 50 minutes. After an introduction that provides the definition of a parabola and connections to other academic subjects and everyday life, the students will be split into four groups that will rotate through four stations. Each rotation will last one class period. One station will have the sheets of paper and pencils for constructing a parabola using a folding exercise. Another station will have the worksheets for constructing a parabola using concentric circles. The other two stations will each consist of a desktop computer with Geometer's Sketchpad installed for dynamic constructions. Since there are sixteen students in each class, each group will have four students and each station will have four chairs. Each student will be engaged in one of four roles utilized at each station: instruction reader, instruction follower, question reader, and question answerer (writes answers to questions on supplied worksheets). Students will switch roles as they rotate from station to station and will fill each role once.

Lesson Structure

I. Introduction

To introduce the lesson, the basic properties of a parabola as the graph of a quadratic equation are reviewed. Students are told that there is another way to view a parabola, using geometry. The parabola is then reintroduced as the locus of all points that are equidistant from a point and a plane. Since this lesson involves finding the parabola from a point and a line, properties of a line are reviewed, focusing on how to find the distance from a point to a line. The following question is then posed: "How can we find a point that is equally distant from a fixed point and a line?" Students are given an opportunity to make conjectures that are written on the board. The students are instructed to keep these ideas in mind as they progress through the lesson tasks. The tasks will be introduced along with the steps required to complete the tasks. Each group will be handed a packet of four pages, each page having the steps and questions to be answered at the end of each task.

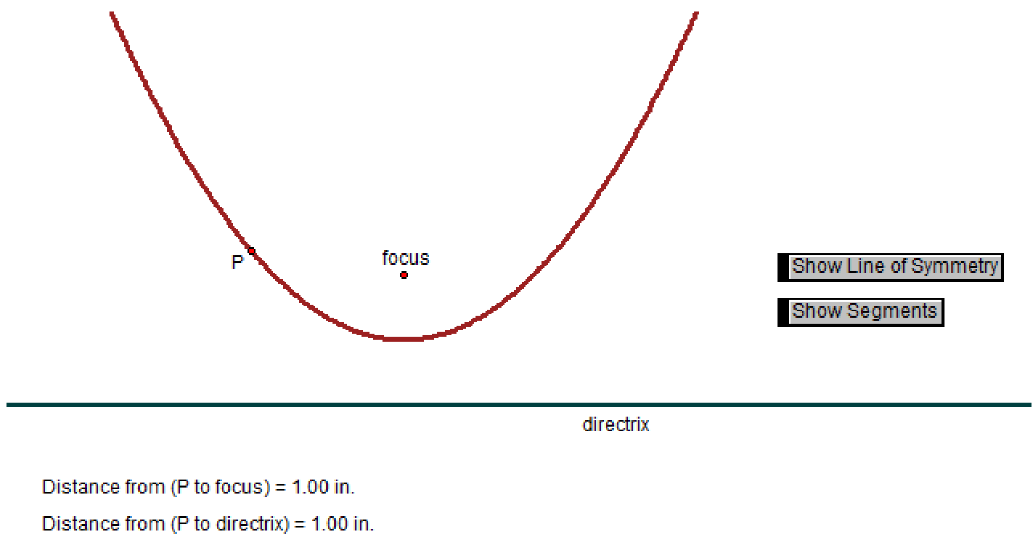
The review of the basic properties of lines, points, and parabolas as graphs of quadratic equations ensures access to all students, since the students, by the time they have reached Math III, have had multiple exposures to these topics in Math I and Math II. Cognitive demand is maintained throughout the task by focusing students' attention on procedures that are utilized to develop a better understanding of parabolas. By following the step-by-step procedures and by answering the questions that follow them, connections are established between students prior knowledge and the underlying concepts involved with understanding parabolas. The tasks result in graphical representations of parabolas that, in later discussions, will lead to algebraic representations that are similar to and different from previously covered algebraic representations, with greater awareness of the connection between the two. The questions that follow the procedures force students to engage with the ideas and concepts so that the task can be completed successfully and understanding is created.

II. Investigation

During the investigation, each group goes to the assigned starting station. The students in the group are allowed to pick the role they will assume in their starting station, with the understanding that each group member must fill a different role at each station until each member has filled all four roles. This change of roles will help to keep students engaged and minimize frustration with the task. Whenever possible, the teacher will have extra steps to follow to extend the activity for students who finish the activity quickly. During the investigation the teacher will monitor group work, only joining the groups and posing questions when needed, as well as making notes to select student responses for whole class discussion. Since the students are following "*procedures with connections*" tasks, steps are followed during the investigation phase that lead to solution strategies which are shared and discussed during the Whole Class Discussion Phase.

The tasks for each station are as follows:

<u>Station 1</u> uses a desktop workstation with Geometer's Sketchpad (GSP) installed on it. A pre prepared GSP sketch is opened on the desktop which shows a parabola along with its focus and directrix.



The sketch also contains two measurements showing the distance of point P on the parabola from the focus and from the directrix. The students are directed to drag point P and notice that the distance measurements change, but always remain equal to each other.

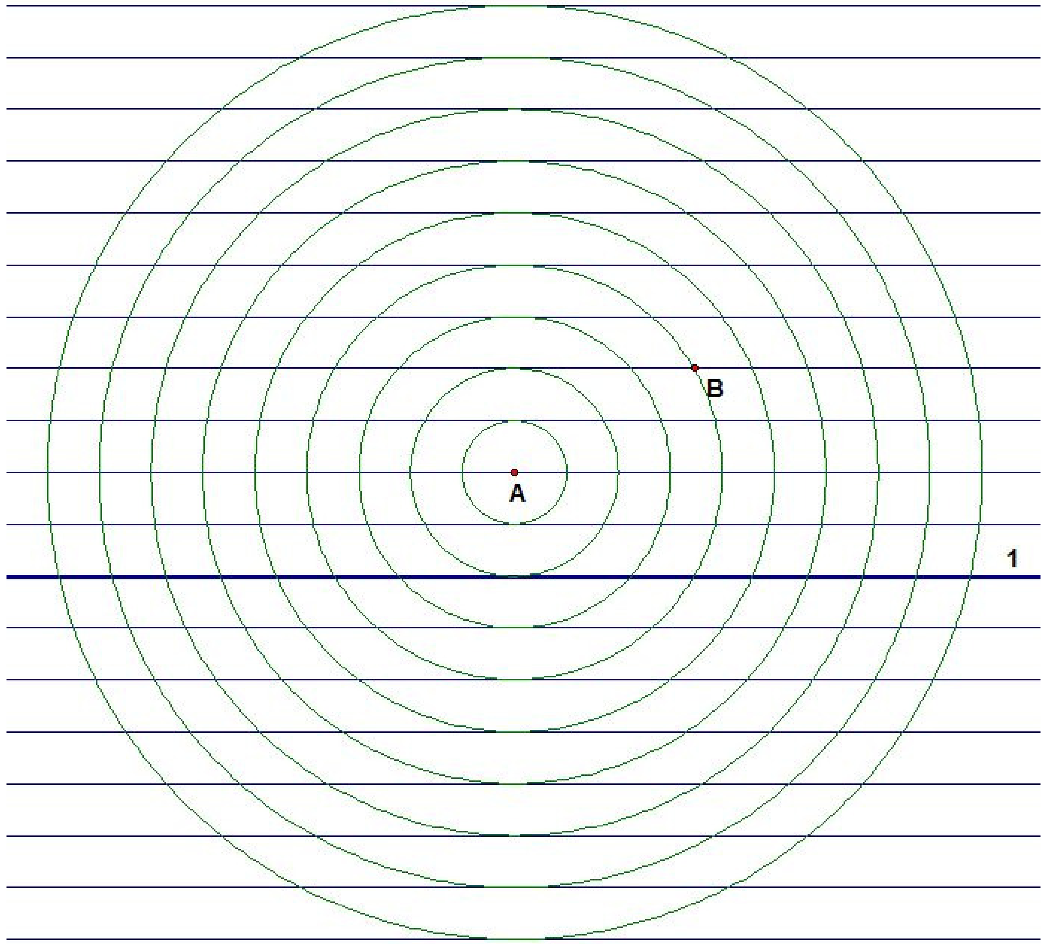
The following questions are asked at Station 1:

- Given a line d and a point P not on the line, how do we define the distance between them?
- By pressing the "Show Segments" button, segments are displayed that represent the distance from point P to the focus and from point P to the directrix. Sketchpad measures the lengths of these two segments. What do you expect to find as point P is dragged along the parabola?
- Given just the parabola's focus and directrix, how can you construct its vertex?

Possible Solution Strategies for Station 1

Given the review on distance from a point to a line, the students should be able to connect the properties in the review with the definition of the distance from point P to the directrix. Since the segments representing the distances from P to the focus and to the directrix are displayed when pressing the "Show Segments" button, students should be able to see that as P is dragged along the parabola, the two distances remain the same, even though the distances themselves do change. Also, since the students have worked with parabolas previously as graphs of quadratic functions in Math I and Math II, they should know that the vertex is either the minimum or maximum value of the parabola, the top or bottom of the curve. Since the vertex in the sketch is at the bottom, students should be able to see that the vertex is the midpoint of the segment representing the distance from the focus to the directrix.

Station 2 uses the sheets of paper with concentric circles printed on them.



There are nine concentric circles centered at point A. The radii of the circles each increase by one unit all the way up to nine units. The horizontal lines are also spaced one unit apart. Each line (except the line passing through point A) is tangent to one of the nine circles. This arrangement of circles and lines is used to draw a parabola.

The following questions are asked at Station 2:

- How many units apart are points A and B? How many units apart are point B and line 1? What can you conclude, based on these two measurements?
- Locate and mark at least 15 points, that sit on a parabola with focal point A and line 1 as its directrix. How did you find these points?

- Using these points, sketch the parabola.

Possible Solution Strategies for Station 2

Students must take care to use the circles as representations of distance from point A and the horizontal lines as representations of distance from line 1 (the directrix). As long as the students can realize that the intersection points of equal unit distance horizontal lines (from line 1) and circles (from Point A) are on the parabola, then the points should be relatively easy to find.

There is a possible difficulty that may arise with students who have visual perception problems. These visual perception problems can cause students to count the incorrect number of circles or horizontal lines, resulting in the students choosing incorrect intersection points. This can be alleviated by having students slowly count circles and lines and mark circles and lines of equal distance with colored pencils.

Station 3 utilizes the sheets of regular 8 ½ x 11" paper. The instruction follower, the question reader, and the question answerer at the station will each take one of the sheets of paper and mark a point A about one inch from the bottom of the paper, centered on the paper. The paper will be folded so that a point on the bottom edge of the paper touches point A and a crease in the paper is made. The paper is unfolded and the process is repeated so that another point along the bottom of the page touches point A. After several creases are folded (about a dozen), unfold the paper and examine to locate any patterns. After resuming the paper folding activity, a well defined curve should begin to take shape. Discuss and compare the folded curves of your group partners and yourself.

The following questions are asked at Station 3:

- The creases seem to form the outline of a parabola. Where do the focus and directrix appear to be?
- If you move point A closer to the bottom edge of the paper and fold another curve, how do you think its shape would compare to the first curve?

Possible Solution Strategies for Station 3

If students are folding the paper correctly, then the creases should form a parabola around point A. Point A represents the focus of the parabola and the bottom of the page represents the directrix. As point A is moved closer to the bottom edge of the paper, if students are not forming the creases again, then student responses could be an expectation of either a skinnier or flatter curve. The answer to this question can be visualized better if students form creases around the newly located point A or by the GSP sketch that follows in Station 4.

Station 4 uses a desktop workstation with Geometer's Sketchpad (GSP) installed on it to model the paper folding exercise performed at Station 3. A new blank sketch is opened and a horizontal line is drawn near the bottom of the screen using the line tool. Point A is drawn above the line centered between the left and right sides of the screen. Another point B is constructed on the line. The crease is formed by constructing a segment from A to B, finding the midpoint of that segment, and constructing a perpendicular bisector of segment AB. As point B is dragged along the horizontal line, the crease adjusts to point B's movement. Selecting the crease line and choosing "Trace Line" from the Display menu allows students to see the collection of creases as point B is moved along the horizontal line. By using the "Locus Tool" the traces can be constructed in a way to adjust with a relocation of point A. Now a better idea of the solution is apparent for question 2 from Station 3.

The following questions are asked at Station 4:

- How does the appearance of the curve change as you move point A closer to the horizontal line?
- How does the appearance of the curve change as you move point A away from the horizontal line?
- (Extension) How does the appearance of the curve change as you move point A below the horizontal line?

Possible Solution Strategies for Station 4

Since the activity at Station 4 is similar to the activity at Station 3, the results should be similar. The difference between the two activities lies in the tool being used to show a dynamic representation of the paper folding activity. As point A is moved closer to and away from the horizontal line, the crease marks automatically adjust to reflect the changes resulting from relocating A.

III. Whole Class Discussion of Investigation

Once all four stations have been visited by all four groups, the students will return to their own assigned seats in the class for a discussion of the findings. A representative from one of the four groups (the group is preselected during teacher's monitoring of the activities) will come to the overhead (ELMO document camera) and demonstrate the group findings. The teacher will ask the selected group for the reasoning behind their findings. The teacher will also ask the other groups if their findings were similar or the same and will ask them to explain the reasoning behind different outcomes. This process will repeat, beginning with Station 1 and ending with Station 4.

After the findings from each activity are discussed, questions are posed to make sense of the findings and connect them to the properties of parabolas, expand on the ideas suggested from the findings, and begin to form generalizations from the patterns that are observed.

Questions posed from the Station 1 Activity are:

- When moving the thumbtacks closer together and further apart from each other, two extremes were observed. What shape did you trace when the thumbtacks were moved apart and the string was taut? Why did your trace resemble a line?
- When both ends of the string were tied to the same thumbtack, what shape did you trace? Why did your shape resemble a circle?

Questions posed from the Station 2 Activity are:

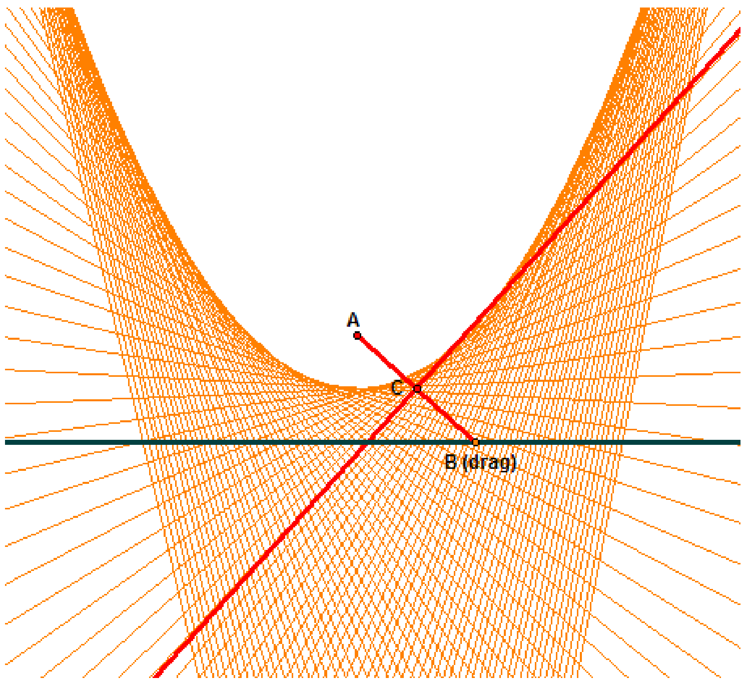
- When point C was dragged between points A and B, a relationship between the radii of the two circles was observed. What was that relationship? How did you determine that relationship?

Questions posed from the Station 3 and 4 Activities are:

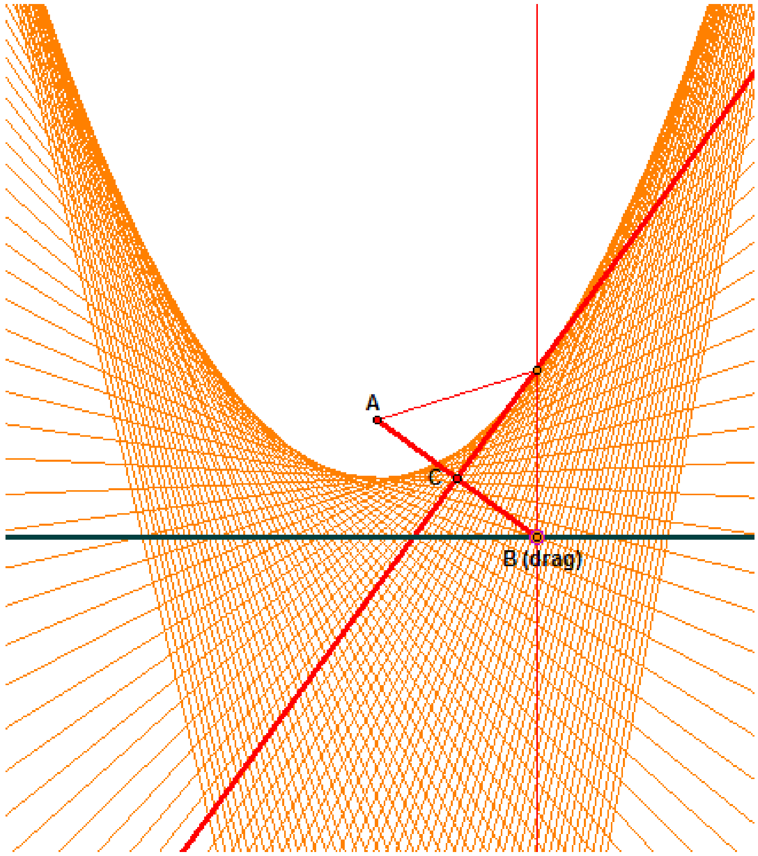
- When point B was merged with the circle, what similar result was observed from the Station 1 Activity using thumbtacks and string? What caused these results?
- When point B was merged with the center, what similar result was observed from the Station 1 Activity? What caused these results?
- Did anyone move point B outside of the circle? What happened to the crease/Locus pattern? What shape did it form?

IV. Closure and Extension

To wrap up discussion and close the lesson, the key properties of a parabola are summarized through analysis of conjectures posed during the introduction, applications of the parabola are discussed, including projectile motion and construction applications, and an assessment is used which is based on a final class activity. We use the Promethian Board and open a premade GSP sketch that shows a thick crease line and its locus.



By dragging point B, it is observed that the crease line is tangent to the parabola. The exact point of tangency is at the intersection of the crease line and another line that is not shown above. We can find the point of tangency by constructing a line perpendicular to the horizontal line and passing through point B.



The following question is asked as a Ticket out of the Door:

- If the intersection point is on the parabola, which two segments must you prove to be equal in length?

References

Scher, D. (2002). *Exploring conic sections with the geometer's sketchpad*. (pp. 3-13). Emeryville, CA: Key Curriculum Press.

Conics & sketchpad . (2011, February 15). Retrieved from http://www.youtube.com/watch?v=X3HhpS3m0Cw