The University of Georgia

Mathematics Education Program

J. Wilson, EMAT 6600

Areas of a Rectangle

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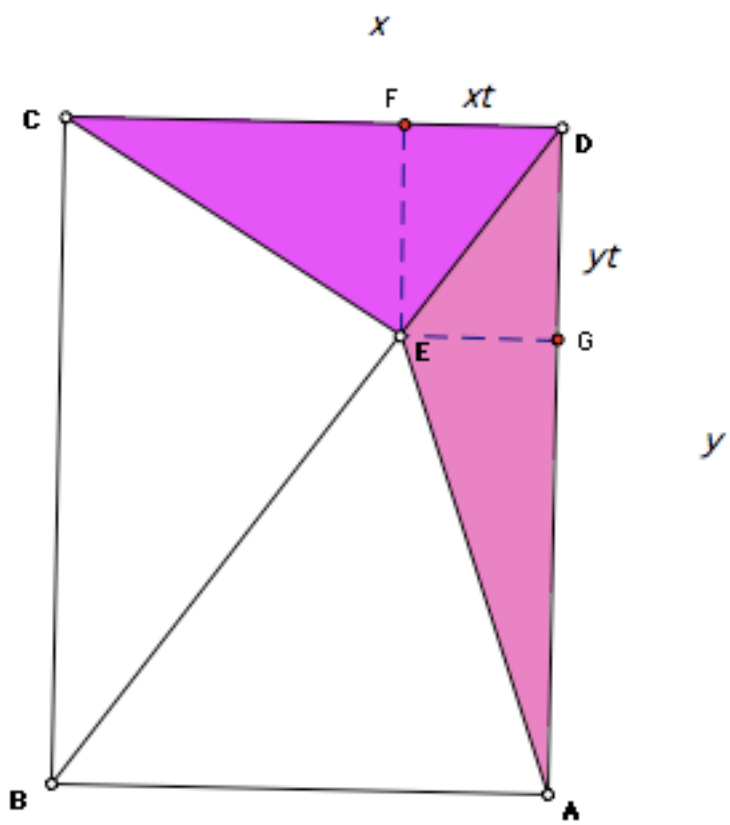
Goal to prove the equality of different areas of a rectangle

Problem

Make Conjectures and proofs for the given sections below:

1) Proof that Area ∆ DEC = Area ∆ DEA



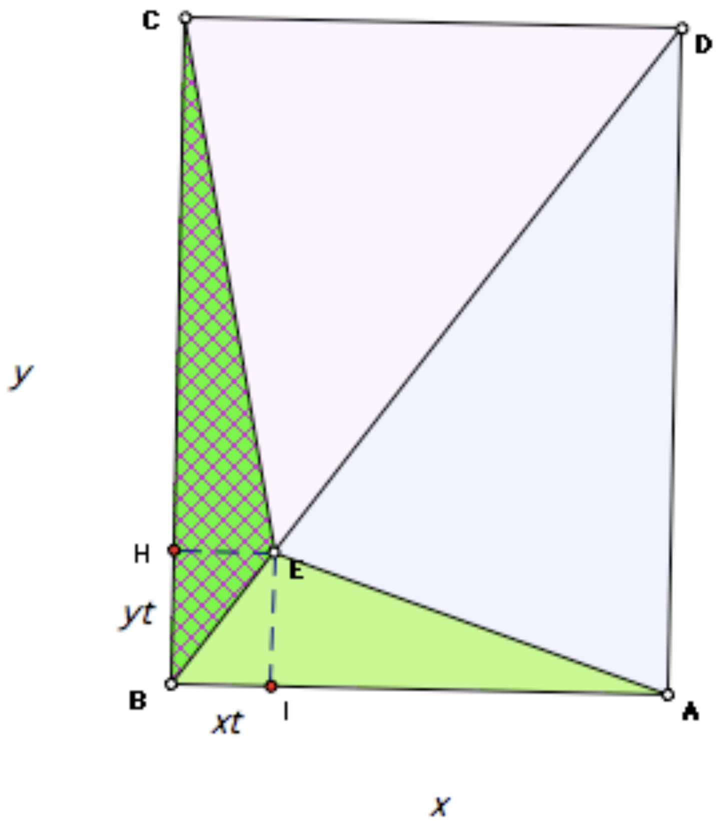
Let length CD = x and FD = xt where xt is the distance along FD corresponding to a given distance DE .Let length AD = y and length GD = yt, where yt is the corresponding distance to a given length DE along the diagonal DB.

The ratio of the segments are x:xt and y:yt and BD:BE are the same because rectangle ABCD is similar to rectangle FEGD by shared diagonal and inscribed rectangle.

Area Δ DEC = $\frac{1}{2}xyt$, Area Δ DEA = $\frac{1}{2}yxt$

Thus both triangles have equal areas.

Proof that Area Δ AEB = Area Δ BEC



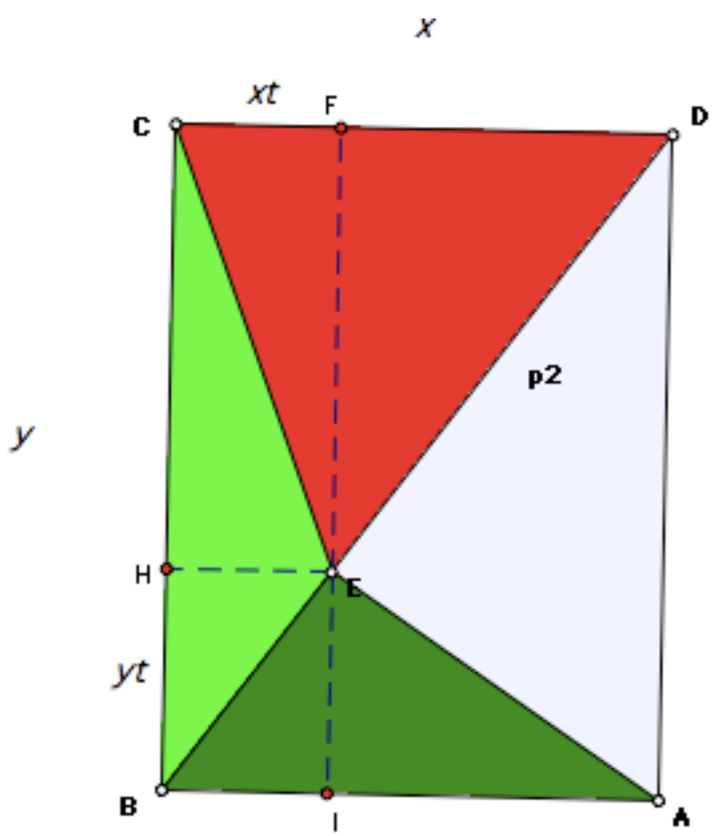
Let length AB = x and IB = xt where xt is the distance along IB corresponding to a given distance DE on the diagonal .Let length CD = y and length HB = yt, where yt is the corresponding distance to a given length DE along the diagonal DB.

The ratio of the segments are x:xt and y:yt and BD:BE are the same because rectangle ABCD is similar to rectangle FEGD by shared diagonal and inscribed rectangle.

Area Δ AEB = $\frac{1}{2}xyt$, Area Δ BEC = $\frac{1}{2}yxt$

Thus both triangles have equal areas.

2) Proof of point where that Area Δ CED = Δ AEB +Area Δ BEC



Let the corresponding parts xt and yt be the similar as marked in question 2.

Area of Δ **CED** = $\frac{1}{2}x(y-yt) = \frac{1}{2}xy(1-t)$

Area of Δ **AEB** = $\frac{1}{2}xyt$

Area of **Δ CEB =** $\frac{1}{2}yxt$

Area of **Δ AEB +** Area of **Δ CEB =** $\frac{1}{2}yxt$ + $\frac{1}{2}xyt$ = xyt

When Area of Area of **Δ CED** = Area of **Δ AEB +**
Area of **Δ CEB :** $\frac{1}{2}xy(1-t)$ = xyt

$\frac{1}{2}(1-t)$ = t

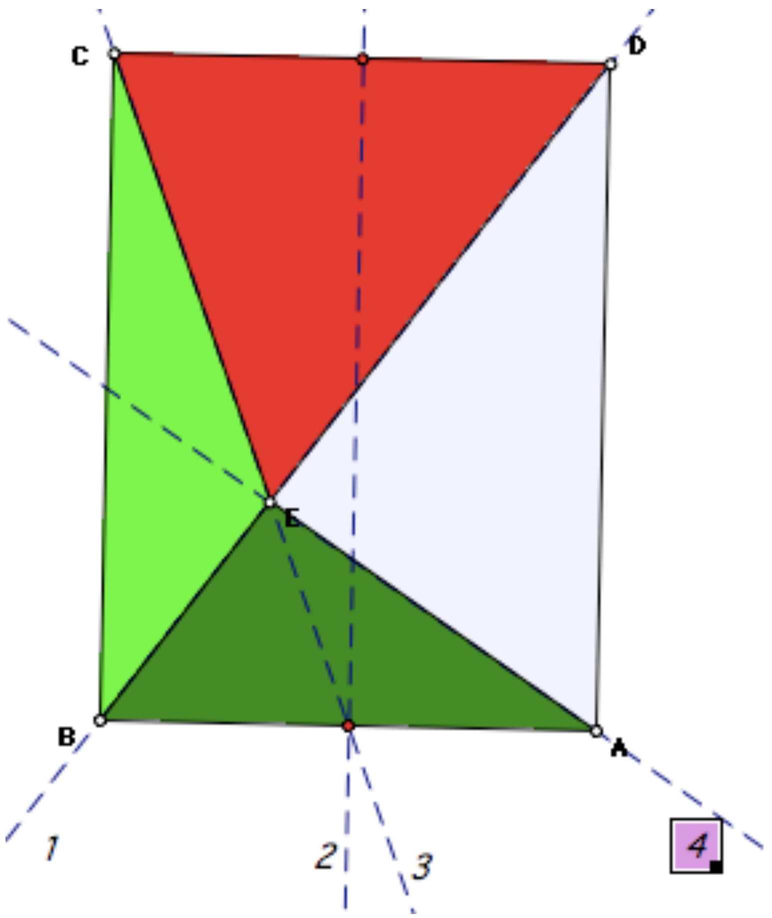
½ - ½ t = t

½ = $\frac{3}{2}t$

t = 1/3

3) Area Δ CED = Δ AEB +Area Δ BEC when point E is 1/3 of the way up along the diagonal from point B.

4) Paper folding Instructions



The paper folding instructions for getting Area Δ CED = Δ AEB +Area Δ BEC are shown as steps 1,2,3 and 4 in the diagram above

fold along the diagonal BD

fold along the midpoints of the short sides of the paper

Fold along the point C and the short side midpoint opposite to C. Let E be the intersection of the folds in steps 1 and 3.

Fold through points A and E.