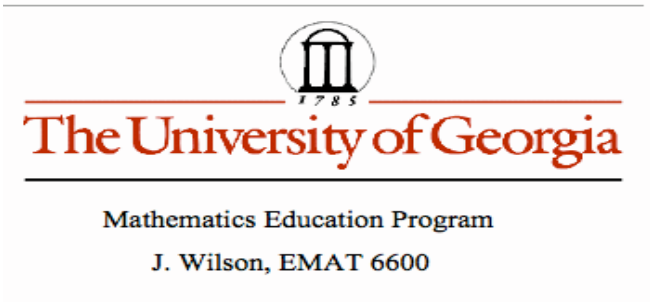


Goat Tethered on Barn

By Leighton McIntyre

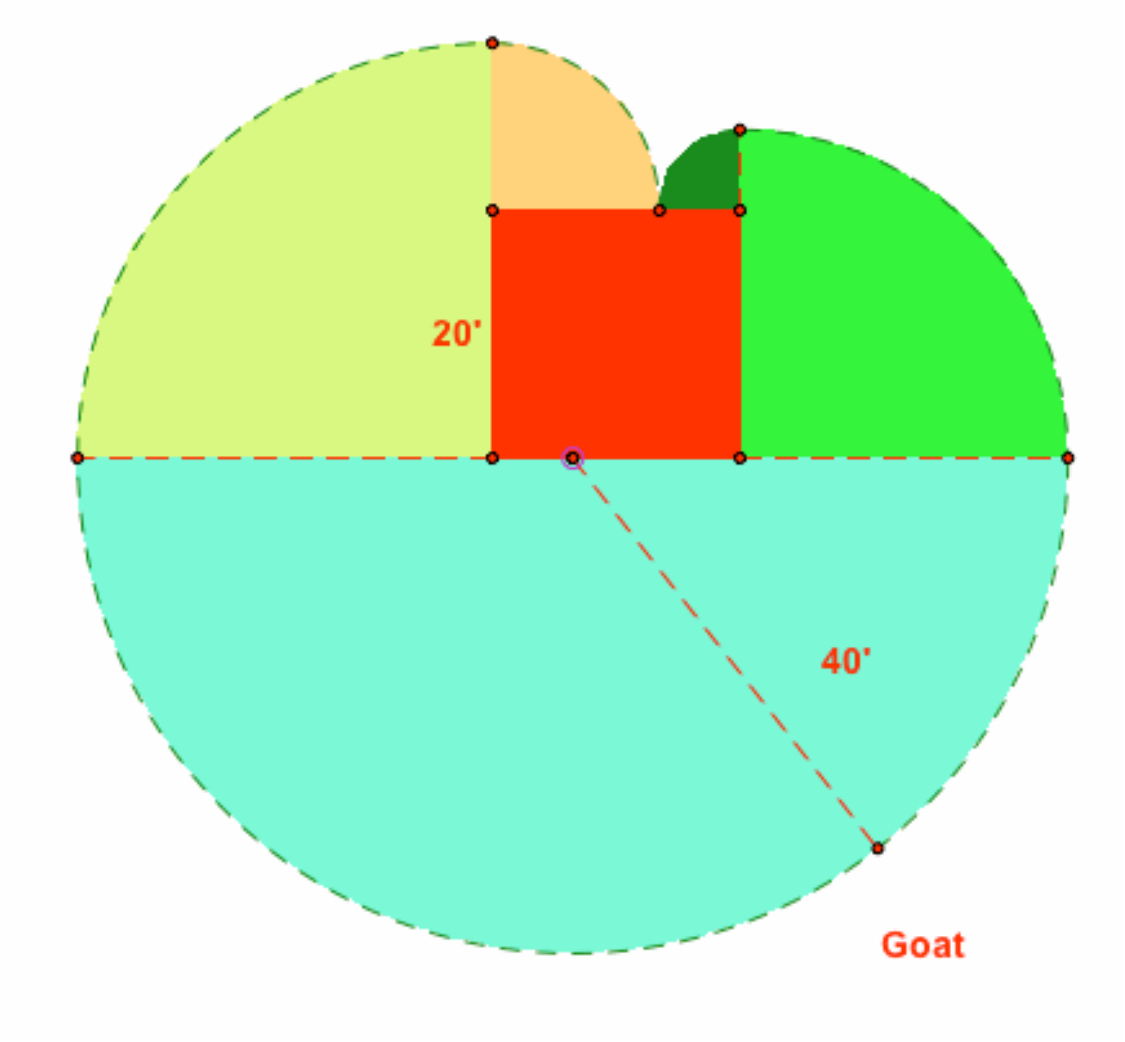
Goal: to find the area of a field a goat can graze that

Problem

Given a square building with each side 20 ft. in a field. A goat is tethered to a point on the base of the building. If the tether is 40 ft. long, what is the area over which the goat can graze?

What if the tether is 50 ft. long?

Does it matter where the tether is tied to the side of the barn? That is, will the area be the same regardless where the tether is connected? Why?



Standard form solution

When the goat moves along its 40 foot rope tethered to the barn, it can sweep out a half circle of radius 40 up the straight line formed with the barn:

Let the radius be $r_0 = 40$

Then $A_0 = ½ \ \pi \ (40)^2$

Let x be the distance from the corner of the barn that the goat is tethered. Then the max length of the rope to the left is 40 – x.

Let r_1 be 40 – x

Then $A_1 = ¼ \pi (40 – x)^2 = ¼ \pi (1600 – 80x + x^2)$

After goat pivots at the top left the max distance is 20 – x.

Let r_2 be 20 – x

Then $A_2 = ¼ \pi (20 – x)^2 = ¼ \pi (400 – 40x + x^2)$

Then the max length of the rope to the right is 20 + x.

Let r_3 be 20 + x

Then $A_3 = ¼ \pi (20 + x)^2$

$= ¼ \pi (400 + 40x + x^2)$

After goat pivots at the top right the max distance is x.

Let r_4 be x

Then $A_4 = ¼ \pi (x)^2$

Finally when the total are grazed by the goat is the sum of the areas

$A = A_0 + A_1 + A_2 + A_3 + A_4$

$= \frac{1}{2}\ \pi\ (\ 40)^2 + \frac{1}{4}\ \pi\ (1600 - 80x + x^2) + \frac{1}{4}\ \pi\ (400 - 40x + x^2) + \frac{1}{4}\ \pi\ (400 + 40x + x^2)$

$+\frac{1}{4}\ \pi\ (\ x\ )^2$

$= \frac{1}{4}\ \pi\ (\ 3200 + 1600 - 80x + x^2 + 400 - 40x + x^2 + 400 + 40x + x^2$

$+ x^2)$

$= \frac{1}{4}\ \pi\ (\ 5600 - 80x + 4x^2)$

$= \ \pi\ (x^2 - 20x + 1400)$

Vertex Form Solution

Use the two end points and midpoint of the side of the barn to which the goat is tethered to locate 3 point on the parabola formed .

Using point 0, 10, and 40 and area formula $A = \pi r^2$

goat tethered at
10

Possible lengths		area grazed
40	1600	800
30	900	450

10	100	50
		1300

goat tethered at 0 AND 40

possible lengths		area grazed
40	1600	1200
20	400	200
		1400

So we have three points on the parabola the minimum (10, 1300π) and the two endpoints (0, 1400π) and (40, 1400π).

Recall the vertex form of a quadratic equation is $f(x) = a(x-h)^2 + k$

Substituting the values (0, 1400π) and (10, 1300π) into the equation gives:

$1400\pi = a(0-10)^2 + 1300\pi$

$100\pi = 100a$

$a = \pi$

So the equation for the area formula is

$f(x) = \pi (x-10)^2 + 1300\pi$

If the rope is 50 ft long there will be some overlap of the areas to the top of the barn.

It does not matter what side of the barn the goat is tied to begin with. As it moves around it will eventually graze out all that is within a given radius.

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