***Notes on Discovering the Pythagorean Identities Learning Task***

***This task launches the unit by introducing the concept of an identity in a context that students should be familiar with from Mathematics I. Before students establish identities and use them to solve problems, it is important that they have a good understanding of what an identity is. By substituting the variables in identities with values, students will gain a more concrete understanding of the term identity. Students then use substitution to determine whether an equation is an identity or not. Towards the end, the task leads students through a geometric derivation of the Pythagorean identities. Students then substitute values into these identities as both a review of basic evaluation of trigonometric functions and also as a final opportunity for students to make the idea of an identity more concrete.***

***This task reviews standards MM4A2 and introduces part of standard MM4A5.***

**Launching Task: Discovering the pythagorean identities**

An **identity** is an equation that is valid for all values of the variable for which the expressions in the equation are defined.

You should already be familiar with some identities. For example, in Mathematics I, you learned that the equation is valid for all values of x and y.

1. You will complete the table below by first randomly choosing values for the *x*’s and *y*’s, then evaluating the expressions and . The first row is completed as an example.
   1. Since is an identity, what should be true about the relationship between the numbers in the last two columns of each row?

***Solution: The number in the last two columns of each row should be equal.***

* 1. Complete the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | *y* |  |  |
| **-3** | **2** | **5** | **5** |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

***Solution: Any values could be plugged in for x and y, but for all three rows that students complete, as in the example, the numbers in the last two columns should be equal.***

1. An identity is a specific type of equation. Many equations are not identities, however, because an equation is not necessarily true for all values of the involved variables. Of the eight equations that follow, only four are identities. Label the equations that are identities as such and provide a counterexample for the equations that are not identities.

***Reinforce to students that if a substituted value of a variable results in a valid equation, this does not prove that the equation is an identity. The identity must be true for all values of the variable for which the equation is defined, and since there are an infinite number of values for substitution, establishing that an equation is an identity by substitution is impossible. A good example is in 2e: The equation will be valid for all positive values of x, although any negative value of x provides a counterexample.***

* 1. 

***Solution: Identity***

* 1. 

***Solution: Not an identity. Counterexample:***

***If x = 1, the equation is invalid, because 36 does not equal 26. Note that an infinite number of counterexamples exist, so correct answers will vary.***

* 1. 

***Solution: Identity***

* 1. 

***Solution: Identity***

* 1. 

***Solution: Not an identity. Counterexample:***

***If x = -2, the equation is invalid, because 8 does not equal -8.***

* 1. 

***Solution: Not an identity. Counterexample:***

***If x = 1, the equation is invalid, because  does not equal 2.***

* 1. 

***Solution: Identity***

* 1. 

***Solution: Not an identity. Counterexample:***

***If x = 1 and y = 1, the equation is invalid, because 2 does not equal 1.***

1. In this unit you will investigate several trigonometric identities. This task looks at the Pythagorean Identities, which are three of the most commonly used trigonometric identities, so-named because they can be established directly from the Pythagorean Theorem.

In the figure below, the point (*x*, *y*) is a point on a circle with radius *c*. By working with some of the relationships that exist between the quantities in this figure, you will arrive at the first of the Pythagorean Identities

*y*



**

*c*

*a*

*b*

(*x*, *y*)

*x*

***It is important that students understand that this is a general figure of a triangle with angle in standard position and that the relationships derived from reasoning about this figure would be valid no matter where the position of  is in the figure. This generality is what allows us to conclude that the resulting identity is valid for all values of .***

* 1. Use the Pythagorean Theorem to write an equation that relates *a*, *b*, and *c*.

***Solution: ***

* 1. What ratio is equal to ?

= 

* 1. What ratio is equal to ?

= 

* 1. Using substitution and simplification, combine the three equations from parts a-c into a single equation that is only in terms of . This equation is the first of the three Pythagorean identities.

***Solution:  from 3b, , and from 3c, ***

***By substituting these expressions into ,***

***We get , which simplifies as follows:***

******

***=***

***=***

1. Since the equation from 3d is an identity, it should be true no matter what  is. Complete the table below, picking a value for  that is in the appropriate quadrant. Use your calculator to round values to the nearest hundredth if the angle you choose is not found on the unit circle. How can you use this data to verify that the identity is valid for the four values of  that you chose?

***Solution: Answers will vary, since there are an infinite number of values to plug in for. One row is completed as an example. If the last column in each row is 1, then the data verifies that the identity is valid for the chosen values of .***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | \* |  |  |
| QI |  |  |  | 1 |
| QII |  |  |  |  |
| QIII |  |  |  |  |
| QIV |  |  |  |  |

\*, , and so on. This is just a notational

convention mathematicians use to avoid writing too many parentheses!

1. The other two Pythagorean identities can be derived directly from the first. In order to make these simplifications, you will need to recall the definitions of the other four trigonometric functions:

   

* 1. Divide both sides of the first Pythagorean identity by  and simplify. The result is the second Pythagorean identity.

***Solution:  ***

***=***

* 1. Divide both sides of the first Pythagorean identity by  and simplify. The result is the third and final Pythagorean identity.

***Solution:  ***

***=***

1. Since the equations from 5a and 5b are identities, they should be true no matter what  is. Complete the table below, picking a value for  that is in the appropriate quadrant. Use your calculator to round values to the nearest hundredth if the angle you choose is not found on the unit circle. How can you use this data to verify that identities found in 5a and 5b are both valid for the four values of  that you chose?

***Solution: Again, answers will vary, since there are an infinite number of values to plug in for. One row is completed as an example. If the values in the third and fourth columns are equal in each row, then the data verifies that the identity involving tangent and secant (from 5a) is valid for the chosen values of . If the values in the fifth and sixth columns are equal in each row, then the data verifies that the identity involving cotangent and cosecant (from 5b) is valid for the chosen values of .***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
| QI |  |  |  |  |  |
| QII |  |  |  |  |  |
| QIII |  |  |  |  |  |
| QIV |  |  |  |  |  |