***Notes on The Sum and Difference Identities Learning Task***

***In this task, students derive the sum identity for the sine function, in the process reviewing some of the geometric topics and ideas about proofs learned in Mathematics I. This derivation also provides practice with algebraic manipulation of trigonometric functions that include examples of how applying the Pythagorean identities can often simplify a cumbersome trigonometric expression. Note that only one of the sum and difference identities is derived. Derivations of the other three sum and difference identities that are part of the GPS are much simpler and would be good activities for students in need of additional challenges.***

***After students have derived one of the identities, they then apply all four of the identities in the context of evaluating trigonometric functions that are not multiples of 30 or 45 degrees. Rewriting expressions in order to solve trigonometric equations is one of the more common applications of the sum and difference identities. This application is not covered in this unit, as it will be addressed in the unit that covers solving trigonometric equations.***

***This task introduces more of standard MM4A5.***

**The sum and difference identities**

In this task, you will use the sum and difference identities to solve equations and find the exact values of angles that are not multiples of  and . Before you apply these identities to problems, you will first derive them. The first identity you will prove involves taking the sine of the sum of two angles.



We can derive this identity by making deductions from the relationships between the quantities on the unit circle below.

****

1. Complete the following congruence statements:
   1. 
   2.   
   3. By the ***SAS*** congruence theorem,  
   4.  
2. Write the coordinates of each of the four points on the unit circle, remembering that the cosine and sine functions produce x- and y- values on the unit circle.
   1. R = ( ,  )
   2. Q = ( ,  )
   3. P = (***1, 0***)
   4. S = (, )
3. Use the coordinates found in problem 2 and the distance formula to find the length of chord .

***Solution: ***

***=squaring each binomial***

***= rearranging terms***

***= applying a Pythagorean identity***

***=***

1. **a.** Use the coordinates found in problem 2 and the distance formula to find the length of chord .

***Solution:***

******

***=squaring each binomial***

***=rearranging terms***

***=applying a Pythagorean identity twice***

***=***

**b.** Two useful identities that you may choose to explore later are  and . Use these two identities to simplify your solution to 4a so that your expression has no negative angles.

***Solution:***

******

***=***

***=***

1. From 1d, you know that . You can therefore write an equation by setting the expressions found in problems 3 and 4b equal to one another. Simplify this equation and solve for . Applying one of the Pythagorean Identities will be useful! When finished, you will have derived the angle sum identity for sine.

***Solution:***

******

*** squaring both sides***

*** adding 2 to both sides***

*** dividing both sides by 2***

The other three sum and difference identities can be derived from the identity found in problem 5. These four identities can be summarized with the following two statements.





Recall that so far, you can only calculate the exact values of the sines and cosines of multiples of and . These identities will allow you to calculate the exact value of the sine and cosine of many more angles.

1. Evaluate  by applying the angle addition identity for sine and evaluating each trigonometric function:



***Solution:***

***==***

1. Similarly, find the exact value of the following trigonometric expressions:
   1. 

***Solution: ***

* 1. 

***Solution: ***

* 1. 

***Solution: ***

* 1. 

***Solution: ***