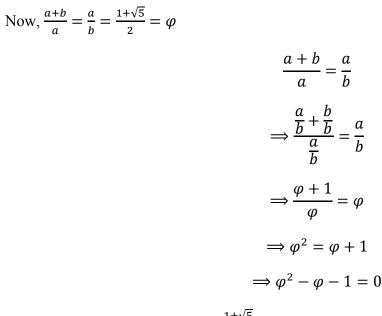# Sublime Triangle Problem

Problem: The *Sublime Triangle* is an isosceles triangle with angles of measure 36, 72, and 72 degrees. It is the only triangle with angle measures in the ratio 1:2:2.

Prove that the ratio of a lateral side of a sublime triangle to its base is the golden ratio,

$$\varphi = \frac{1 + \sqrt{5}}{2}$$

Solution: Since $\angle ABC \approx \angle ACD$; $\angle BAC \approx \angle CAD$ and side AC is common. We have $\triangle ABC$ and $\triangle ACD$ are similar isosceles triangles.

Now, $\frac{a+b}{a} = \frac{a}{b} = \frac{1+\sqrt{5}}{2} = \varphi$

$$\frac{a + b}{a} = \frac{a}{b}$$

$$\implies \frac{\frac{a}{b} + \frac{b}{b}}{\frac{a}{b}} = \frac{a}{b}$$

$$\implies \frac{\varphi + 1}{\varphi} = \varphi$$

$$\implies \varphi^2 = \varphi + 1$$

$$\implies \varphi^2 - \varphi - 1 = 0$$

Applying quadratic formula, $\varphi = \frac{1+\sqrt{5}}{2}$.