

A Tangled Tale Problem

Problem: A man walked for 5 hours, first along a level road, then up a hill, and then he turned around and walked back to the starting point along the same path. He walks 4 mph on the level, 3 mph uphill, and 6 mph downhill. Find the distance he walked.

Solution: Given,

Speed at level ground forward/backward = 4 mph

Speed going uphill = 3 mph

Speed going downhill = 6 mph

Let, time travelled uphill = t_1 and time travelled downhill = t_2 .

So, *distance uphill* = $3t_1$ and *distance downhill* = $6t_2$.

Now, he travelled same distance uphill as he did for downhill.

So, $3t_1 = 6t_2 \Rightarrow t_1 = 2t_2$.

For overall rate going uphill and coming downhill, $r = \frac{3t_1+6t_2}{t_1+t_2} = \frac{6t_2+6t_2}{2t_2+t_2} = \frac{12t_2}{3t_2} = 4$.

So, average rate travelling up and downhill is 4 mph, which is equal to average rate travelling level ground.

Hence, total distance covered in 5 hrs is 20 miles.

We get same average speed for level ground as we do for hills. This happens due to the average speeds for up and downhill are 3 mph and 6 mph respectively. Different values, however, will result average speed not equal to average speed of level ground. In fact, it would be impossible to solve (i.e. find the total distance) for values other than 3 mph and 6 mph.