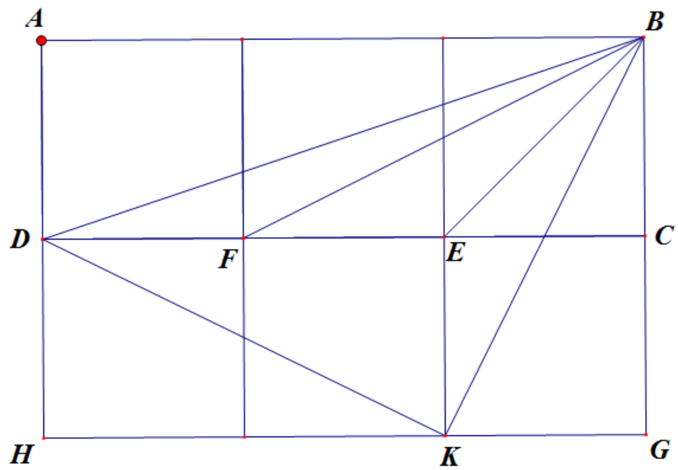
Arctan problem

Problem: Provide a geometric proof/demonstration that

$$\arctan\left(\frac{1}{3}\right) + \arctan\left(\frac{1}{2}\right) = arctan(1)$$

Solution: We start with three unit squares (stacked side-by-side) and make a square ABCD. We can easily prove that $\angle BDC + \angle BFC = \angle BEC$ (the proof is left out here on purpose).



Now, $\angle BEC = 45°$. So, $\angle BDC + \angle BFC = 45°$

Let's denote $\angle BDC = \alpha$ and $\angle BFC = \beta$

So, $\alpha + \beta = 45°$

From the diagram, for right triangle $\Delta BDC, \tan(\alpha) = \frac{1}{3}\ and \tan(\beta) = \frac{1}{2}$.

Using trigonometric identity,

$$\tan(\alpha+\beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\cdot\tan(\beta)}$$

$$= \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}}$$

$$= 1$$

So, $$\tan(\alpha+\beta) = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\tan(\alpha+\beta) = 1$$

$$\alpha + \beta = \arctan(1)$$

$$\arctan\left(\frac{1}{3}\right) + \arctan\left(\frac{1}{2}\right) = \arctan(1)$$

This follows from a general trigometric Diophantine equation:

$$\arctan\left(\frac{1}{x}\right) + arctan\left(\frac{l}{y}\right) = arctan\left(\frac{1}{k}\right)$$

Where k and l are positive integer such that gcd $(l\ ,\ k^2 + 1) = 1$ (or l and $K^2 + 1$ are relatively prime).

(The general equation and proof comes from an article here)