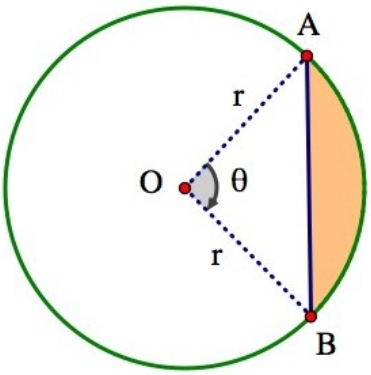
Area of a Segment of a circle

Problem: Develop the formula for the area of Segment of a circle with radius r where the arc of the circle segment subtends a central angle of θ.



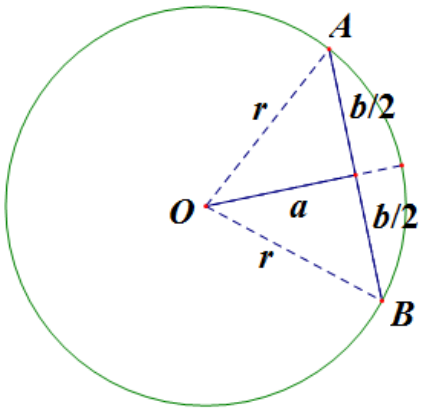
Solution: To find the area of the shaded region, we have to perform the following:

Area of the shaded region = Area of the arc sector – Area of the triangle.

Now, the area of the arc generated by central angle θ is

$Area_{arc} = \frac{1}{2}\theta r^2$, where θ is in radians.

To find the area of the triangle ΔOAB, draw an angle bisector a. Hence we have two right triangles. Using trigometric identities,



$$sin\left(\frac{\theta}{2}\right) = \frac{\frac{b}{2}}{r} \Rightarrow b = 2rsin\left(\frac{\theta}{2}\right)$$

And $\cos\left(\frac{\theta}{2}\right) = \frac{a}{r} \Rightarrow a = rcos\left(\frac{\theta}{2}\right)$

Now, area od triangle $\Delta OAB = \frac{1}{2} \cdot b \cdot a$

$$= \frac{1}{2} \cdot 2rsin\left(\frac{\theta}{2}\right) \cdot rcos\left(\frac{\theta}{2}\right)$$

$$= \frac{1}{2}\, r^2 \left(2sin\left(\frac{\theta}{2}\right) cos\left(\frac{\theta}{2}\right)\right)$$

$$= \frac{1}{2}\, r^2 \sin(\theta)$$

So, $Area_{segment} = \frac{1}{2}\theta r^2 - \frac{1}{2} r^2 \sin(\theta)$, where θ is in radians.