Astroid Equations

Part I: Show that the two equations $x^{2/3} + y^{2/3} = 1$ and $(x^2 + y^2 - 1)^3 + 27x^2y^2 = 0$ produce same graph algebraically.

Proof: I will show that the first equation yields the second,

$x^{2/3} + y^{2/3} = 1$ ……………………….(equation 1)

$(x^{2/3} + y^{2/3})^3 = (1)^3$

$x^2 + 3x^{4/3}y^{2/3} + 3x^{2/3}y^{4/3} + y^2 = 1$ …………….(equation 2)

$x^2 + y^2 - 1 = -3(x^{4/3}y^{2/3} + x^{2/3}y^{4/3})$

$(x^2 + y^2 - 1)^3 = -27(x^{4/3}y^{2/3} + x^{2/3}y^{4/3})^3$

$= -27(x^4y^2 + 3x^{\frac{10}{3}}y^{\frac{8}{3}} + 3x^{\frac{8}{3}}y^{\frac{10}{3}} + x^2y^4)$

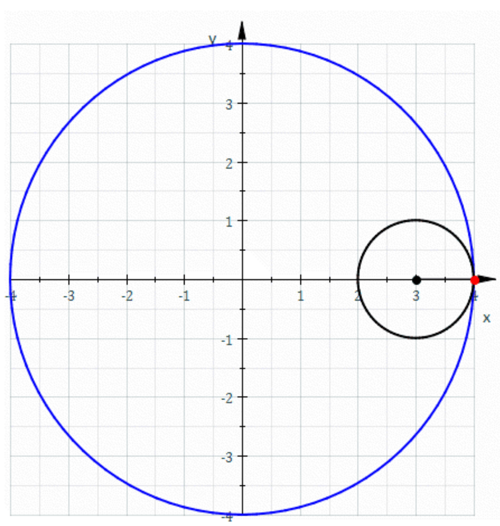
$= -27x^2y^2(x^2 + 3x^{4/3}y^{2/3} + 3x^{2/3}y^{4/3} + y^2)$

$= -27x^2y^2$ ………………….(using equation 2)

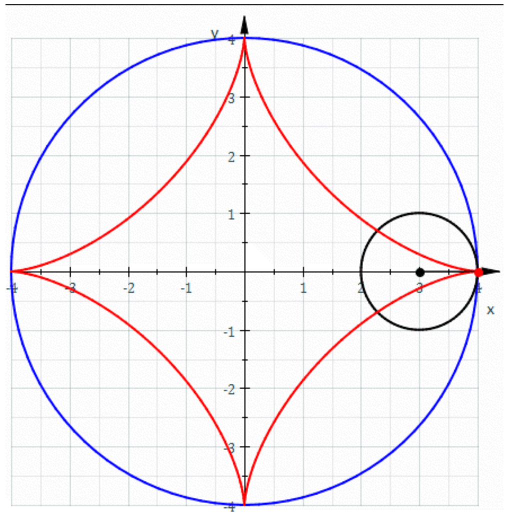
So, $(x^2 + y^2 - 1)^3 + 27x^2y^2 = 0$.

Part II: Build a GSP sketch to simulate a small circle of radius r rolling around the inside of a larger circle of radius R, where R = 4r.

Below, we have a large circle with radius 4 and inscribed a smaller circle with radius 1.



If we rotate the smaller circle inside of the larger circle and trace the locus of a point on the rim of the small circle as it rolls, we get an Astroid (picture below)



The GSP file could be found here: Astroid