

Bottles and Cans Problem

Problem: Three neighbors named Quincy, Penny, and Rosa took part in a local recycling drive. Each spent a Saturday afternoon collecting all of the aluminum cans and glass bottles he or she could. At the end of the afternoon each person counted up what he or she had gathered, and they discovered that even though Penny had collected three times as many cans as Quincy and Quincy had collected four times as many bottles as Rosa, each had collected exactly the same number of items, and the three as a group had collected exactly as many cans as bottles. Added together, the three collected fewer than 200 items in all. How many cans and bottles did each collect?

Solution: Let x, y, z be the number of bottles collected by Quincy, Penny and Rosa respectively. And a, b, c be the number of cans collected by Quincy, Penny and Rosa respectively.

Let, n be the total number of bottles and cans collected by each person.

So, we have,

$$n = a + x = b + y = c + z$$

$$\text{Now, } x = 4z \Rightarrow n - a = 4(n - c) = 4n - 4c \Rightarrow c = \frac{1}{4}a + \frac{3}{4}n \dots\dots\dots(i)$$

$$\text{And, } b = 3a \Rightarrow n - y = 3(n - x) \Rightarrow y = 3x - 2n \dots\dots\dots(ii)$$

We know that the group collected as many cans as bottles, so we have:

$$a + b + c = x + y + z$$

$$\text{So, } a + 3a + \frac{1}{4}a + \frac{3}{4}n = x + 3x - 2n + \frac{1}{4}x$$

$$\frac{17}{4}a + \frac{3}{4}n = \frac{17}{4}x - 2n$$

$$17a + 3n = 17x - 8n$$

$$11n = 17(x - a) = 17(x - n + x) = 17(2x - n)$$

$$11n = 34x - 17n$$

$$34x = 28n$$

$$x = \frac{14}{17}n$$

$$\text{Hence, } x = \frac{14}{17}n$$

$$a = \frac{3}{17}n$$

$$b = 3a = \frac{9}{17}n$$

$$c = \frac{1}{4} \cdot \frac{3}{17}n + \frac{3}{4}n = \frac{27}{34}n$$

$$y = n - b = n - \frac{9}{17}n = \frac{8}{17}n$$

$$z = \frac{1}{4}x = \frac{1}{4} \cdot \frac{14}{17}n = \frac{7}{34}n$$

Since all fractions are in their lowest forms, hence n must be divisible by 34 (the LCD of all fraction).

All together they collected less than 200 bottles. So individually they collected $\frac{200}{3}$ bottles and cans. Since number of bottle and cans cannot be fraction, the nearest whole number that is divisible by 34 is 34. So, $n = 34$.

So, Quincy collected 28 bottles and 6 cans; Penny collected 16 bottles and 18 cans; and Rosa collected 7 bottles and 27 cans.