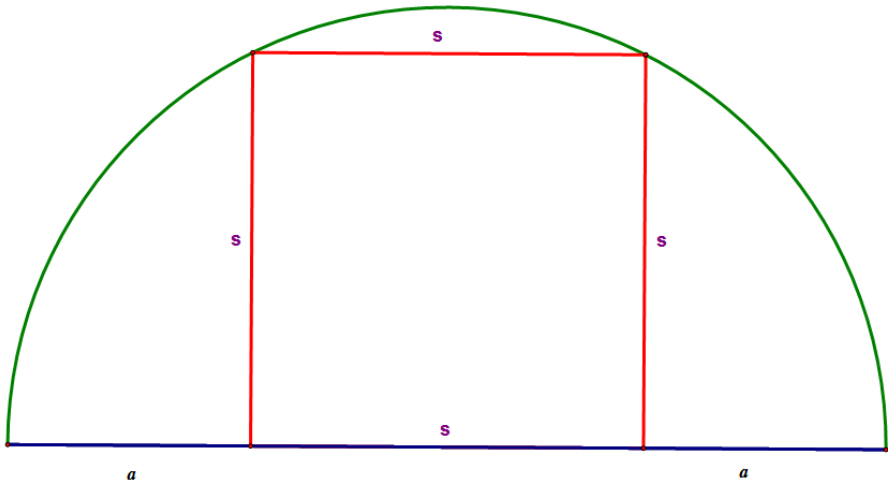
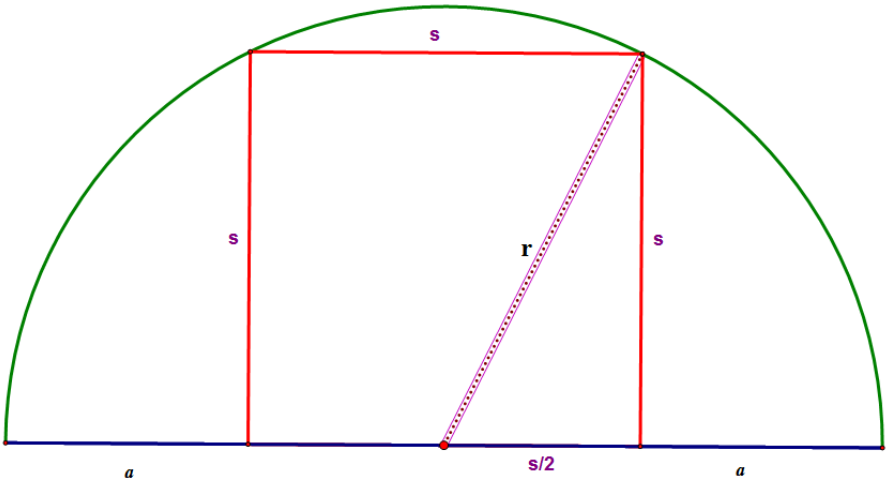
Square Inside a Semicircle

Problem: Given a semicircle with an inscribed square of side s. Let a be the length of the diameter on each side of the square. We want to find the ratio of s to a. What is $\frac{s}{a}$?

Solution: The above problem could be translated into the diagram here



Now, let's join a vertex of the square and the center of the circle. The segment is the radius of the semicircle.



As a result, we have a right triangle. Here the radius of the semicircle,

$r^2 = s^2 + \left(\frac{1}{2}s\right)^2$ ……..(using Pythagorean formula)

$$r^2 = s^2 + \frac{1}{4}s^2$$

$$r^2 = \frac{5}{4}s^2$$

Hence, $r = \frac{\sqrt{5}}{4}s$. However, $r = \frac{s}{2} + a$ as well. Setting them equal to each other, we get

$$\frac{\sqrt{5}}{2}s = \frac{s}{2} + a$$

$$a = \frac{\sqrt{5} - 1}{2}s$$

So, $\frac{a}{s} = \frac{\sqrt{5}-1}{2} = \varphi$ (the Golden Ratio!!!).