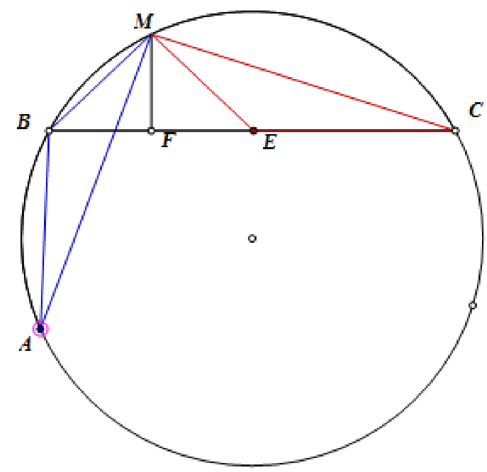
Proof of the Broken Chord Theorem

Problem: Let AB and BC make up a broken chord in a circle, where BC > AB, and if M is the midpoint of arc ABC, then the foot F of the perpendicular from M on BC is the midpoint of the broken chord ABC.



Solution: Here I am trying to prove the theorem of the broken chord, which asserts that if AB and BC make up a broken chord in a circle, where BC > AB, and if M is the midpoint of arc ABC, the foot F of the perpendicular from M on BC is the midpoint of the broken chord. Or, AB +BF = FC.

I will construct the proof using congruent triangles and their congruent parts to create an isosceles triangle with point F as the midpoint of the base. On the diagram above, let E be a point on BC such that EC = AB

Since M is the midpoint of arc ABC, we have AM = CM (chords inscribed in congruent arcs are congruent). Additionally $\angle A = \angle C$, since both are inscribed angles intersection chord BM.

Now, with EC = AB, $\Delta ABM = \Delta CEM$ (due to SAS congruence).

Hence BM = ME. Thus we have $\Delta MBE$is isosceles triangle with MF as its altitude.

So, $\Delta BMF = \Delta EMF$(due to Hypotenuse-Leg theorem).

So, we can conclude that BF = FE.

With AB = EC and BF = FE, we get AB + BF = FE + EC.

Since FE + EC = FC, we have our final statement that AB + BF = FC.