The University of Georgia

Mathematics Education
EMAT 4680/6680 Mathematics with Technology
Jim Wilson, Instructor

Exploring the Centroid of a Triangle

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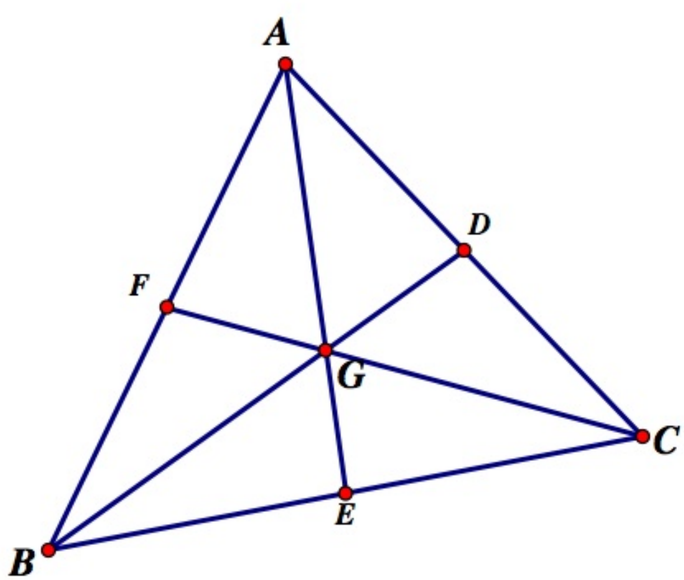
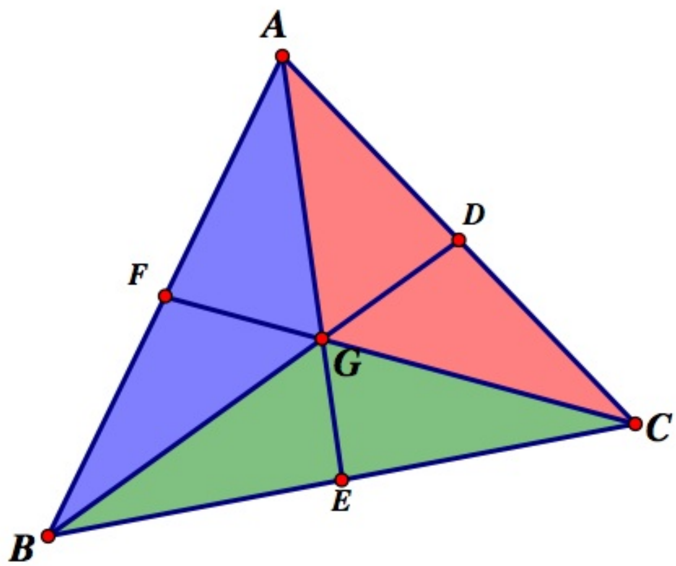
Objective: Define a centroid and show the three medians of a triangle divide the triangle into six small triangles. Show that these small triangles all have the same area.

The centroid is the point of intersection of the triangles three medians. A median is a segment from the midpoint of a side to the opposite vertex. A triangle contains three medians and one centroid. The image below shows a triangle with three medians (labeled D, E, and F) and a centroid (labeled G).



Now we will to prove the claim that the six triangles formed by the centroid have the same area. For the proof we will use the acute triangle representations. However, the argument also holds for obtuse and right triangles.

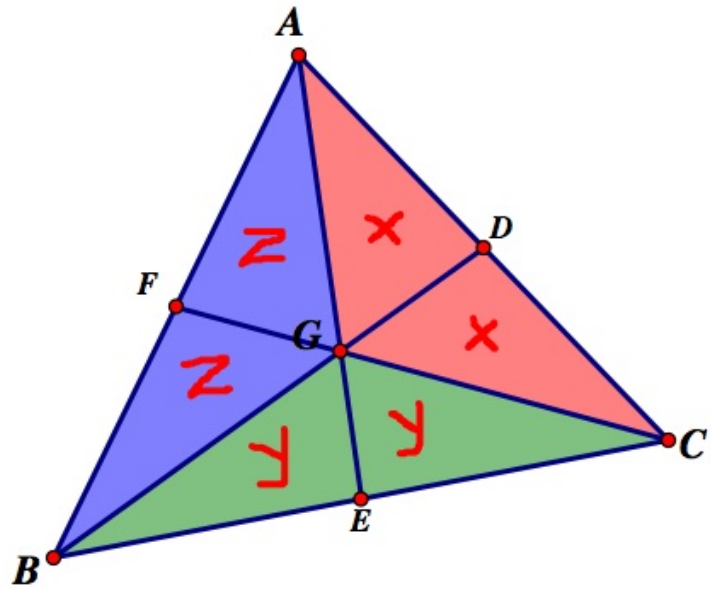
First we will examine three triangles ΔAGC, ΔCGB, and ΔBGA.



We can see that the line segment AD=DC since D is the midpoint of the line segment AC. We can also see that these two triangles share a common side GD, which means they have the same height. Therefore, ΔAGD and ΔDGC have the same area since they have the same base length and the same height length. Similarly, arguments can be made for the two smaller triangles in contained in ΔCGB and ΔBGA.

Now we want to show that the red region, the green region, and the blue region are all equivalent. In order to do this, we will show that ΔAEB and ΔAEC have the same area. We can use similar ideas from above. Since the two triangles share the side AE, we know that they have the same height. We also know that the lengths of their bases are the same since point E is the midpoint of the segment BC. Therefore, ΔAEB and ΔAEC have the same area.

We can now label the small triangle areas with values of x, y, and z, which can be seen visually in the image below.



Since the areas of ΔAEB and ΔAEC are equivalent we can see 2z+y=2x+y, which tells us that z=x. Now, we have four of the small triangles, the red and the blue triangles, that have the same area.

Using similar ideas, we can look at the area of ΔBDC, which is 2y+x, and the area of ΔBDA, which is 2z+x. Since ΔAEB and ΔAEC share the same height and have the same base, AD=DC since D is the midpoint of AC, we can show that they have the same area. Therefore, 2y+x=2z+x, which tells us y=z.

Since z=x and y=z. by the transitive property we can conclude that x=z=y, and all six small triangles formed by the centroid have the same area.