MATHEMATICAL

Putting Umph into

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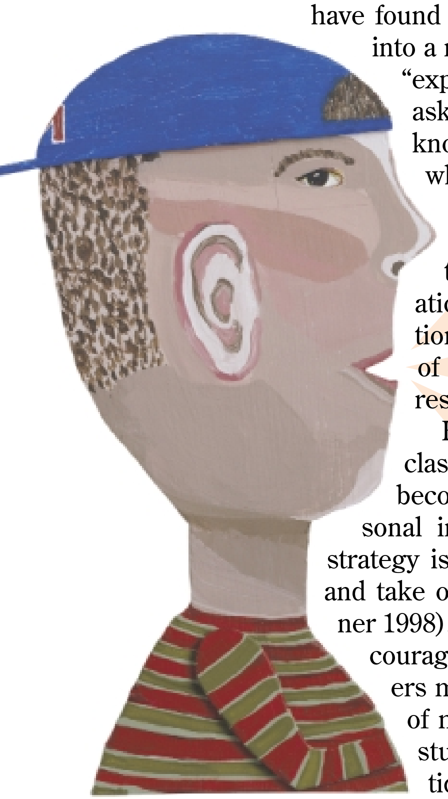
PERHAPS THE MOST DIFFICULT RECOMMENdation of the NCTM's Standards to put into practice is that of orchestrating classroom discourse—moving from a teacher-centered classroom to one that is centered on student thinking and reasoning. Some researchers argue that traditional "chalk and talk" classrooms put all the intellectual authority in the hands of the teacher and little or no responsibility for thinking and reasoning on the shoulders of the students. Classroom discussions, in contrast, are viewed as encouraging students to construct and evaluate their own knowledge, as well as the ideas of their classmates. Few examples or guidelines exist, however, to help teachers orchestrate such discussions.

A prerequisite, of course, is a good task that is rich enough to elicit student thinking and discussion (Smith and Stein 1998; Stein and Smith 1998). Even with good tasks, however, some teachers have found that classroom discussions can fall into a rut. Teachers *always* ask students to "explain their thinking"; students *always* ask one another "why?" and students know that their answers are correct when the teacher stops asking questions (Williams and Baxter 1996). Sometimes, we seem only to have traded the IRE routine (teacher initiation, student reply, teacher evaluation) (Mehan 1979), for a different set of routines, without students' taking responsibility for their own thinking.



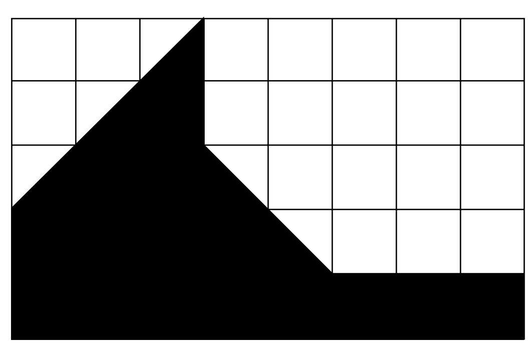
Experts on discourse suggest that classroom discussion does not have to become predictable. Developing a personal interest in a particular solution or strategy is a way that students can invest in, and take ownership of, the discourse (O'Conner 1998). Teachers have the armament to encourage this student interest. First, teachers must create a classroom atmosphere of mutual respect and trust that allows students to feel comfortable in critiquing the work of others and in risking making mistakes themselves. Second, teachers should select instructional tasks that prompt students to take different positions and find different solutions. Finally, they can encourage students to align with a position and to defend that position, convincing others of its correctness with mathematical evidence.

A Provocative Task

THE SAME TASK WILL NOT PROVOKE THE SAME degree of investment in discussion in every middle-grades classroom. As the teacher, you are the best judge of whether a particular task will lead your students to take different positions. The task shown in **figure 1** contains the seeds to produce disagreement. Do your students understand how to compare the linear and area measurements of figures with similar units? If not, have them try the task in **figure 1.**

This activity will work best if you have an overhead projector; transparencies, including one with the problem on it; markers; and measurement tools for students, such as rulers and square grids. Place the problem on the overhead projector, and ask the students to work on it for a few minutes, then to share their answers and reasoning strategies with partners. Tell them that you will select certain students to present their answers and strategies to the class.

Find the area of the shaded region in square centimeters and square millimeters.



□ = 1 square centimeter

Fig. 1

ARGUMENTATION:
Classroom Discussions

Discussion of the Task

WALK AROUND THE ROOM, AND ASSESS THE STUdents' solutions and reasoning. In a typical middle school class, most students will have no problem determining that the shaded region has 17.5 square centimeters. A range of answers for the number of square millimeters will probably be given, however. Many students will erroneously multiply 17.5 times 10 to arrive at the incorrect answer of 175 square millimeters. Others will realize that the 17.5 square centimeters must be multiplied by 100 because 100 square millimeters are in every 1 square centimeter; these students will arrive at the correct answer of 1750 square millimeters. If both these answers and strategies are represented in your class, you have the makings of a good mathematical argument!

In the next phase of the lesson, students share their responses with the whole class. After establishing that the shaded area is 17.5 square centimeters, you must next decide which solution to the query about square millimeters to begin with, the incorrect one or the correct one. I suggest that you start with the answer 175 square millimeters. As a student presents this solution, your role is not to control or evaluate. Rather, you need to make sure that the student's reasoning is clear to the rest of the class, including his or her justification for multiplying by a factor of 10. If necessary, paraphrase the student's reasoning to clarify his or her thinking for the class to evaluate. Then open the discussion to the class with a question, such as "Does everyone agree with ____? If not, I should see your hand up, ready to ask a question."

With this invitation, you are signaling the students that they have a responsibility to *(a)* listen closely to the first student's reasoning and to try understand it and *(b)* formulate a critique of the answer if they do not agree with it. If students have a different answer, they should be able to relate their own approaches to the displayed approach, showing where their thinking diverges, and be able to articulate why their thinking is correct.

As students respond to the incorrect solution, you should allow the student who offered that solution to respond to the critiques and clarify his or her reasoning processes. You should also encourage other students who have solved the problem in the same way to agree publicly. You can acknowledge these students with a statement, such as "Jerlyn, Robert, and Samuel agree with Teresa's approach; they say that you need to multiply by 10 because we used that approach when we changed centimeters to millimeters earlier in the year." You must align students with the claim and give them responsibility for defending it. As the teacher, your job is to set up the positions of the mathematical argument, not to step in and adjudicate it.

At some point, you must bring the correct solution, fully explained and justified, into the discussion. You might say, for example, "Others of you are still not convinced and think that this problem can be solved in another way. Who would like to come to the overhead and show us a different way?" Then the burden is on the other side both to present an alternative strategy and to convince classmates that the alternative approach is the correct one. Students who have been aligned with the first position are, of course, given the same opportunity to critique the second solution. Remember that the students, not you, have the responsibility to defend their claim and that those who dispute the claim, not you, have the responsibility to find the flaws in their reasoning. Your job is to clarify the poles of the debate and to animate students to stand for or against the different approaches.

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"Take Time for Action" encourages active involvement in research by teachers as part of their classroom practice. Readers interested in submitting manuscripts pertaining to this theme should send them to "Take Time for Action," MTMS, NCTM, *1906 Association Drive, Reston, VA 20191-9988.*

Summary of Research on Mathematical Argumentation

SEVERAL COLLEAGUES AND I EXTENSIVELY ANAlyzed a middle school mathematics lesson that featured this task (Forman et al. 1998). In the preobservation interview, the teacher indicated that she was struggling with becoming less directive in her role as a teacher. Although she had once believed that success breeds success, she had grown to realize that she had an unfortunate tendency to spoon-feed her students. At the time of this lesson, she was trying to break that pattern.

The first solution displayed was the incorrect one, in which square centimeters were multiplied by 10 instead of 100. Orchestrating a debate around this solution was complicated, however, by the fact that the student had also dropped the .5 from the square centimeters before multiplying by 10. This mistake caused some consternation among his classmates and was an interesting sidebar to the discussion, but the error was not the focus of the debate that the teacher was attempting to initiate. Soon, however, another student directly challenged the first student about why he had multiplied by 10. After a few exchanges related to the challenge, two different students modeled correct solutions. The second of these solutions—complete with a grid—even explained the relationship between considering two dimensions of a square, its length and width, to find its area and using the procedural tactic of moving the decimal two places. Throughout this argument, students supplied mathematical evidence and reasoned logically to back up their positions. The role of the teacher was to encourage risk taking and alignment with one position or the other in the debate and to paraphrase and clarify student discussion.

In our research, we used constructs from the fields of sociolinguistics and rhetoric to analyze the transcript of this discussion. We found that the student discussion followed many of the processes of effective argumentation put forth by rhetoricians, such as examining premises, using warrants to back claims, and presenting counterarguments effectively. In addition, we observed that students were more likely than the teacher to initiate explanations, to provide answers or claims backed by appropriate justifications, and to evaluate their own and one another's arguments.

Equally important, we were able to analyze and explain the teacher's success in actively involving students in explaining their ideas, listening to one another, and evaluating their own and others' arguments. We found that the teacher participated by recruiting attention and participation from the class and by aligning students with positions through rephrasing their contributions; highlighting their positions through repetition; and pointing out implicit but important aspects of their explanations through expansion, including reminding students to say "*square* millimeters."

Implications for Teaching and Learning

OUR RESEARCH WAS CONDUCTED UNDER THE auspices of the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project (Silver and Stein 1996) and took place in predominantly poor middle schools that served students with diverse racial and linguistic backgrounds. We believe that this research provides an existence proof that middle school students from low-income, urban neighborhoods can engage in collective argument, which many theorists and educators believe is important to creating a community of mathematical and scientific practice. Our research also illustrates that teachers—with judicious selection of tasks and coaching to prompt student participation in their solution—can break out of the role of sole evaluator of student thinking and reasoning in the classroom.

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