1785

The University of Georgia

A simple proof of why a circle is not squareable might be this:

Let the circle have a radius of 1. Then the area of the circle is $\pi 1^2 = \pi$. Therefore in order for the circle to be quadrable then $\pi = x^2$ and $x = \sqrt{\pi}$. Since π is not constructible by straightedge and compass then the circle is not quadrable.

Lindemann, in 1882, was able to prove that circles are not quadrable. He used a formula of Euhler's and proofs of Hermite. Euhler using the expansions of various series, e^{ix}, showed that $e^{ix} = \cos x + i \sin x$, and by substituting π for x, $e^{\pi i} = -1$. Hermite proved that in an equation of the form "$a_0 + a_1 e^{p1} + a_2 e^{p2} + ... = 0$ the exponents and coefficients are not only not integers but that they cannot all be algebraic numbers." (Bold) Lindemann was able to show in the equation $e^{\pi i} + 1 = 0$, since 1 and i are algebraic then π must be transcendental. This means π cannot be a root of a polynomial equation. Therefore circles are not quadrable.

(Bold)