The Mathematical Education of Prospective Teachers of Secondary School Mathematics: Old Assumptions, New Challenges[1]

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We address two questions: What mathematics do prospective secondary school mathematics teachers need to know? In what context should they come to know it? Consideration of both matters has implications for the revision of the undergraduate program in mathematics.

What mathematics do prospective secondary school mathematics teachers need to know?

Teachers must know the mathematics they teach. Deciding exactly what this means, and then determining what more mathematics they need, are not simple matters. Typically, two perspectives have influenced the design of programs for the preparation of secondary teachers, and both are relevant to mathematics departments:

1. Prospective high school teachers should study essentially whatever *mathematics majors* study—because this will best equip them with a coherent picture of the discipline of mathematics and the directions in which it is heading, which should influence the school curriculum.
2. Prospective high school teachers should study *mathematics education*—methods of teaching mathematics, pedagogical knowledge in mathematics, the 9–12 mathematics curriculum, etc.

We argue in this paper that there is substantial knowledge that is necessary for effective teaching but which is neglected by this two-pronged approach. Furthermore, much of this knowledge is mathematical in character, and, as such, should be a responsibility of mathematics departments. Because this knowledge is particular for the teaching of mathematics, it lies, in a sense, between mathematics education and traditional undergraduate mathematics content. Keep in mind, however, that there is much outside of mathematics and mathematics education that all secondary school teachers need to know, about students, about learning, about teaching, about curriculum, and about the contexts of schooling.

History of recommendations[2]

The dominant approach to the mathematical preparation of secondary school teachers in the United States in recent years is to require that they complete an undergraduate major (or a near-equivalent) in mathematics.

[1] The authors wish to thank Deborah Loewenberg Ball, Dick Stanley, Tom Rishel, Merle Heidemann, and Dawn Berk for their comments and assistance in preparing this paper.
[2] For more detail on the following history, see Gibb, Karnes, & Wren, 1970.

Interestingly enough, a quick review of the recommendations in this century about the mathematical preparation of teachers reveals that this trend toward a near-major has generally grown stronger with each set of recommendations. For instance, the 1911 report of the American subcommittee of the International Commission on the Teaching of Mathematics recommended preparation in several areas of pure mathematics, applied mathematics (e.g., mechanics, astronomy, physics), surveying, a "strong course on the teaching of secondary mathematics," other education, and "a course of an encyclopedic nature dealing critically with the field of elementary mathematics from the higher standpoint" (International Commission on the Teaching of Mathematics, 1911, pp. 13–14). There is no explicit call for a major in mathematics. Likewise, the 1935 recommendations of the Mathematical Association of America's Commission on the Training and Utilization of Advanced Students of Mathematics calls for "minimum training in mathematics that goes as far as 6 hours of calculus, Euclidean geometry, theory of equations, and a history of mathematics course" (Commission on the Training and Utilization of Advanced Students in Mathematics, 1935). The courses that might have been more typical of a major at that time (advanced calculus, mechanics, projective geometry, additional algebra) are described as "desirable additional training."

In reports from various groups in the late '50s and early '60s, the expectations for secondary teachers began to sound like a major, with calls for 24 semester hours of mathematics courses (National Council of Teachers of Mathematics [NCTM], 1959), and 30 semester hours, including abstract algebra (American Association for the Advancement of Science, 1959). It was the Committee on the Undergraduate Program in Mathematics (CUPM) that first recommended that "prospective teachers of high school mathematics beyond the elements of algebra and geometry should complete a major in mathematics" (CUPM, 1961). Ten years later, this sentiment was still strongly held: "We regard it as a matter of great importance that a program for teachers should be identical to the one offered to other mathematics majors, except for a few courses peculiarly appropriate to prospective high school teachers" (CUPM, 1971, p. 170).

The 1983 CUPM recommendations do not explicitly call for a mathematics major, but instead list 13 courses, including a 3-course calculus sequence, as the minimal preparation, with a call for additional work for teachers of calculus (CUPM, 1983). It is worth noting that 13 courses is *more* than a major in some institutions. In 1991, the MAA's Committee on the Mathematical Education of Teachers (COMET) assumed responsibility for the preparation of teachers: "These recommendations assume that those preparing to teach mathematics at the 9–12 level will complete the equivalent of a major in mathematics, but one quite different from that currently in place at most institutions" (Leitzel, 1991, p. 27). The recommendations list standards in seven content areas (e.g., geometry, continuous change, and mathematical structures) rather than specific courses.

Since the first CUPM recommendations, most major sets of national committee recommendations offered by the mathematics community and most recommendations from the education community have recommended the equivalent of a major in mathematics as the fundamental preparation for the secondary teacher. Sometimes the recommendation is general and assumes that whatever is considered appropriate as a major is appropriate for future mathematics teachers. For instance, the new recommendations of the National Council of Accreditation of Teacher Education (NCATE), to go into effect next year, expect that candidates for teaching should "know the content of their field (a major or the substantial equivalent of a major)" (NCATE, 2000). The most current recommendations being developed for the mathematical education of teachers (the Conference Board of the Mathematical Sciences [CBMS] Mathematical Education of Teachers project), though reflecting some more current general issues about the undergraduate major, still make the same basic argument, as is evident in the following excerpt:

> The following outline of mathematics and supporting courses is one way to provide core knowledge for future high school teachers while satisfying many requirements in a standard mathematics major.
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> Year One: Calculus, Introduction to Statistics, Supporting Science
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> Year Two: Calculus, Linear Algebra, and Introduction to Computer Science
>
> Year Three: Abstract Algebra, Geometry, Discrete Mathematics, and Statistics
>
> Year Four: Introduction to Real Analysis, Capstone, and Mathematics Education Courses
>
> (CBMS, in preparation)

There is no question that teachers need to know mathematics in order to teach well in secondary schools—the logic in this seems unassailable. Yet at the same time, research studies do not demonstrate a convincing relationship between teachers' knowledge of mathematics (often measured by the number of college mathematics courses taken) and their students' mathematical performance (see Begle, 1979; Monk, 1994). Perhaps teachers fail to learn the content of these courses, or they do learn it but find that it doesn't connect in any recognizable way with their classroom practice.

There are at least two problems with requiring the same preparation for mathematics teaching as for graduate school in mathematics. First, high school teachers are preparing for a professional practice that is completely different from that of conducting mathematical research. The mathematical demands they will face are different. But we are not arguing for less mathematical preparation for teachers. In fact, we would argue that with a typical major in mathematics, teachers may have *too little mathematical preparation of the kind they will need*. Second, by keeping content separate from pedagogy, prospective teachers may fail to acquire what Shulman (1987, p. 8) called *pedagogical content knowledge*—"an understanding of how particular topics, problems, or issues are organized, presented, and adapted to the diverse interests and abilities of learners, and presented for instruction".

The mathematics of the secondary school curriculum

Today's secondary school context is radically different from that of 20 years ago. First of all, secondary schools today take seriously the commitment to educate all students to be prepared for a rapidly changing world—and thus *all* students need to be prepared for the possibility of higher education or a highly technical workplace. This has meant increasing trends away from vocational or general tracks, and toward a foundation of significant mathematics for all students (see the recommendations of the NCTM 2000 *Principles and Standards for School Mathematics*). The range of options today in high school curricular materials reflects this shift. While the algebra 1, geometry, algebra 2, precalculus, calculus sequence that many of us experienced is still alive and well, today's instructional materials also include substantial emphasis on data and statistics, on discrete mathematics, on dynamic geometry, and on early treatment of functions and modeling. Some series are fully integrated, with titles like "math 1, 2, 3, 4," or are completely organized around applications of mathematics and so-called contextual problems. The once unchallenged high school end-goal of advanced placement calculus has given way to other equally strong possibilities, such as advanced placement statistics, or sophisticated technical courses focusing on CAD-CAM technologies, finance, or applications of mathematics to the world of work. These trends are at odds with what has been the traditional mathematics major, with its historic emphasis on abstract algebra and analysis as end-goals. If one takes seriously the notion that being prepared to handle the mathematics of the secondary school is something crucial for teachers, then it seems that these shifts in the nature of secondary school mathematics education need to be taken quite seriously by those who prepare teachers.

For secondary mathematics teachers, it is ironic that, except for occasional concepts that might be called upon in calculus, the entire four years of an undergraduate mathematics major address content that is, on the surface, unrelated to the topics of the high school curriculum. The only place where prospective secondary teachers are very likely to learn about such secondary school topics as the Law of Cosines, the Rational Root Theorem, the proof of Side-Angle-Side congruence, the Zero Product Principle or tests of divisibility is in the secondary school, as students themselves. More substantially, the kinds of integration of mathematical ideas and connections that are necessary in teaching a coherent secondary program, are unlikely to be obvious to students on the basis of their undergraduate program. Consider the following example of a student teacher episode. This student teacher had been a strong undergraduate mathematics major at a small state university; she had taken courses in abstract algebra, geometry at a junior level, and advanced calculus. She was conducting a lesson in algebra 2 class and was presenting the absolute value function. She showed the students the notation, $f(x) = |x|$, and drew the graph. A student said something like the following: "That graph reminds me of angles in geometry. Can we use the absolute value function as a way to write a formula for any angle?" The teacher was completely taken aback by the question. The question would have required the teacher to make a number of mathematical judgments on the spot, and also to connect ideas across content areas in unexpected ways.

As professors working with prospective high school teachers, are we confident that our students will be able to answer the following typical high school students' questions, in ways that are both mathematically sound and also accessible and compelling to a 15 year-old?

- Why does a negative times a negative equal a positive?
- Why do I switch the direction of the less than symbol when I multiply both sides by a negative number?
- In every triangle that I tried in Sketchpad, the angles add up to 180. I don't need to do a proof, do I?
- I am not convinced that .99999… = 1.
- How do I know parallel lines never intersect?
- How do I know that the asymptote never hits the line? I mean, it crosses the line near 0.
- I think that 100 *is* divisible by 3—the answer is 33 and one third.
- Why is it OK to use 22/7 for the value of pi, sometimes?
- I think that the number 1 has three different square roots: 1, -1, and .99999999. I am sure that .99999999 is a square root of 1 because when I multiply it by itself on my calculator I get 1.00000000.

Mathematics for Teaching

Suppose we could construct a curriculum for secondary school teachers that, in terms of mathematical content, was in tune with the current secondary curriculum and its directions of change. Moreover, suppose that it genuinely offered students a chance to see both where the concepts of the high school curriculum are embedded in a larger picture of mathematics and also to see elementary mathematics from an advanced standpoint or to develop "profound understanding of fundamental mathematics" (Ma, 1999). This task probably would require designing some courses especially for teachers, thus breaking with the tradition that what's good for the math major is good for the prospective teacher. Both majors require substantial study of serious mathematics, but there may be reasons why the body of mathematical content is different in some ways.

We suspect there is another body of knowledge that high school teachers also need, which is mostly mathematical in character, and which is probably more within the purview of the mathematics department than it is the school of education. Mathematics education researchers and mathematics educators are struggling with how to describe and talk about this knowledge. Ball and Bass have studied this notion in the elementary grades and call it "pedagogically useful mathematical understanding" (Ball & Bass, 2000). Zalman Usiskin (personal communication) considers "teachers' mathematics" to be a branch of applied mathematics, in the sense that (1) it emanates from the classroom in much the same way that operations research arises from problems in business, and (2) it includes specialized mathematical knowledge that may not be known by mathematicians in other applied or pure areas. In addition to having profound understanding of the content that is taught in the secondary grades, mathematics teachers at this level need to be able to draw on and use other knowledge that is mathematical in character, such as:

- finding the logic in someone else's argument or the meaning in someone else's representation;
- deciding which of several mathematical ideas has the most promise, and what to emphasize;
- making and explaining connections among mathematical ideas;
- situating a mathematical idea in a broader mathematical context;
- choosing representations that are mathematically profitable; and
- maintaining essential features of a mathematical idea while simplifying other aspects to help students understand the idea.

These kinds of mathematical activities, we would argue, are essential in teaching. They arise while teachers are planning lessons, designing tasks for class and for assessments, interacting directly with students around content, answering their questions, and correcting their work. If such activities are essential, then prospective teachers need to learn the necessary mathematical skills—ideally, we contend, through mathematical material that is close to the material of high school classrooms, under the guidance of mathematicians in university mathematics departments.

There are at least three approaches for creating connections between undergraduate and high school mathematics:

A mathematical approach. Prospective teachers should study high school mathematics from an advanced standpoint.[3] The approach is to find and exploit topics in the high school curriculum that can be extended and elaborated in ways that are sophisticated mathematically. (See Box 1 for a list of principles that guide this approach.) An alternative is to find topics in the typical undergraduate curriculum and look for ways to connect them with key areas of the high school curriculum—this is the idea behind capstone courses or "shadow courses" that prospective teachers take alongside such courses as abstract algebra. Although these approaches may represent an improvement over some mathematics courses that have no connection to high school mathematics, the approach runs the risk of leaving prospective teachers without sufficient pedagogical content knowledge, which lies at the intersection of content and pedagogy.

An integrative approach. Integrate the goals of the mathematics content and pedagogy courses so that teachers might be better able to see connections and later use them (see, e.g., Cooney et al., 1996).

An emergent approach. Analyze the practice of teaching and determine what mathematical knowledge teachers draw upon in their practice. Then use real mathematical "problems" of mathematics teaching practice as sites for learning mathematics, taking advantage of the mathematical opportunities that emerge while working on the problems.

We believe that all of these approaches are worth pursuing. The emergent approach, however, is the most unusual and requires the most explanation. To understand this approach, it is useful to characterize the traditional approach as a failure at helping teachers transfer their mathematical knowledge into practice. Rather than constructing a solution apart from teaching, the approach begins in the context of teaching practice. We try to identify the interpreting, problem-solving, and decision-making activities in which a teacher actually engages, so that we may infer what mathematics is actually used. The next step would be to design a curriculum around such activities, in much the same way one might create a mathematics curriculum for engineers or social scientists by looking at the mathematical problems they have to solve.

Mathematics education researchers and professional developers (see, e.g., Ball & Cohen, 1999; Schifter, Bastable, & Russell, 1999; Shulman, 1992; Stein, Smith, Henningson, & Silver, 2000; Barnett, Goldenstein, & Jackson, 1994) have begun to explore this approach through the use of videos of classrooms, student work, written cases, and student curriculum materials. The notion of using the actual work of teaching as a starting point for thinking about the mathematical preparation of teachers was explored further at the Teacher Preparation Mathematics Content Workshop hosted in 1999 by the Mathematical Sciences Education Board (National Research Council, 2000). The ideas are just beginning to take shape in the mathematics education community. Future development will require some concerted work in conceptualizing the emergent approach more fully and designing experiences in which prospective teachers might profitably acquire this mathematical knowledge.

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[3] The University of California at Berkeley together with the University of Chicago School Mathematics Project (UCSMP) are major participants in a grant, awarded by the Stuart Foundation of San Francisco, to create and test a new type of college mathematics course described as "High School Mathematics from an Advanced Standpoint." Principal writers for the materials are Dick Stanley of the University of California at Berkeley, UCSMP Director Zalman Usiskin of the University of Chicago, Anthony Peressini of the University of Illinois, and Elena Marchisotto of California State University, Northridge. Initial pilot testing is underway.

[4] We wish to acknowledge the original work and thinking of Deborah Ball and Hyman Bass in advancing this particular line of argument.

Box 1. Principles for extended problem analysis (Stanley & Callahan, in progress)

The mathematical content involved in extended analyses of problems can be expressed as a set of mathematical "principles." They include:

1. Selecting parameters to represent key quantities in a problem situation. Typically the parameters replace numerical values of some of the quantities that are used in the initial statement of the problem. There are many subthemes here, such as:
   a. considering which quantities to parametrize;
   b. being alert for ways to generalize the results being found, and at the same time looking for important special cases;
   c. replacing a variable x that has a particular range $0 < x < L$ with a proportional variable p with a range $0 < p < L$.
2. Coaxing expressions into their most useful forms. Again, there are many subthemes:
   a. collapsing separate occurrences of the independent variable;
   b. making use of ratios in particular and dimensionless factors in general.
3. Representing relationships in a situation in several different kinds of ways to get different insights. Examples are diagrams, graphs, tables, formulas.
4. Looking for connections to different kinds of mathematics. For example:
   a. looking for geometrical interpretations of analytic results, and conversely;
   b. looking to connect discrete mathematics with continuous mathematics.
5. Anticipating, asking, and answering many of the sorts of questions that may occur to a reader who is trying to understand the ideas. Standard treatments often bypass such questions since they are not part of the most efficient and elegant presentation.

It is worth pointing out that this approach, while affording some new opportunities, is not without potential pitfalls. One potential pitfall is that discussion among current or future teachers might remain mired in school mathematics and fail to move toward higher mathematics. Another is that most of the discussion might be spent on more general teaching and learning issues. Our experience in presenting these ideas to a variety of audiences, however, is that with a well-chosen example the discussion can be deep and substantive, often leading to mathematical territory that is unexplored or implicit in the traditional curriculum.

Conclusions

Some of the current challenges in secondary school mathematics education today, coupled with new insights from research in mathematics education, suggest that it may be time to move away from the little questioned assumption that has historically guided mathematics education in the past several decades—that a major in mathematics, or something that deviates from it only marginally, is the best mathematical preparation for prospective teachers of secondary school. Teachers need to understand mathematics deeply. They must understand its applications and how ideas are integrated across subject matters. And they must be able to see mathematical possibilities in students' statements or written work. Could new majors be designed specifically for prospective secondary school mathematics teachers, that bring together the three kinds of mathematical knowledge described here, in ways that would serve our secondary teachers well? Given that as many as half of the mathematics majors, at least in some research universities, intend to become high school teachers, such development is warranted. The recommendations of the Mathematical Education of Teachers draft (CBMS, in preparation) are timely in their recognition of the need to diversify the offerings in the undergraduate curriculum for the prospective secondary teacher. Increasingly, the set of recommended offerings diverges from what at least has been the mathematics major. We need to prepare teachers to solve the kinds of mathematical problems that actually arise in teaching. This kind of thinking has not prevailed for secondary school teachers in mathematics.

In What Context Should Prospective Teachers Come to Know Mathematics?

The responsibility for the mathematical content preparation of secondary school mathematics teachers has been, historically, the responsibility of departments of mathematics—and this should continue to be the case, in our view. Some rather serious issues need to be confronted, however, if departments are to provide effective preparation. Within the faculty, who is responsible for the mathematical content knowledge of secondary teachers? What expertise do faculty members need, and how do they acquire it? Do they need to be working in schools, conferring with secondary teachers, and staying current in their knowledge of mathematics education? And, if their primary background is in mathematics education, how do they remain current in mathematics?

Departmental environment

Little research has been conducted about the student learning environment for mathematics majors who intend to teach secondary school mathematics, although there is much anecdotal information available. A frequent complaint among these students is that their experience in mathematics courses, in particular the nature of the mathematics instruction, is inconsistent with what they are learning in their education courses about the best ways to help students learn. In education courses about pedagogy, where they may be learning about the importance of actively engaging their future students, finding ways to make the subject matter meaningful and to connect it to other concepts, building on what students know, and using embedded assessments that call for explanation, they are developing certain knowledge and images about what effective mathematics teaching is like. In many upper-level mathematics courses, however, the instruction does not include these elements. So, despite the fact that these upper-level majors often are good mathematics students and have succeeded within the system, they sometimes are very conflicted about what should happen in their own mathematics teaching, because of the variation and dissonance they experience.

The use of technology in the undergraduate experience for prospective teachers is also an issue. NCTM and state standards recommend the use of technology in secondary schools to support mathematics learning. Thus, prospective teachers need experience as learners in using technology in advanced mathematical settings. Some institutions offer “reform” calculus or technology-rich calculus courses to students in the life sciences, or in some engineering and science tracks, but not to students majoring in mathematics, and therefore not to prospective secondary school teachers. In this case the teachers, as mathematics majors, take the more theoretical, less applied calculus option and don’t see the applications, connections, or experience the role of technology.

There are also aspects about the student learning environment (outside the classroom) that are problematic for strong students who have expressed interests in secondary school mathematics teaching. In conversations with their faculty advisors and mentors, such students report that sometimes mathematician advisors discourage them from teaching, with arguments about seeking more lucrative and prestigious options. Yet across the country, and especially in some urban areas, substantial numbers of middle school and secondary school children are being taught mathematics by teachers without even the equivalent of a minor in the field. It follows that without good preparation of students in mathematics in high school, the pressure to offer remedial courses at the undergraduate level and the lack of a supply of strong students into mathematics will continue to be problems facing higher education. Undergraduate mathematics faculty should be eager to encourage good and interested students into mathematics teaching.

A second difficult aspect of the student environment is related to advising. Students intending to be teachers need to meet the requirements of their major, as well as a set of course requirements and clinical experience requirements in professional education. Teacher education students typically have very full and demanding programs. Taking courses in the correct sequence at the correct stage of their undergraduate career is crucial to staying on track. With increasing numbers of teacher education programs being fifth-year programs, or five-year programs, or combined bachelors and masters degree programs, issues about when to apply to the program and deadlines for registering for student teaching, for internships, and for various state and national exams, become crucial to students' ability to complete their programs. Mathematics department advisors need to be aware that advising mathematics education students is complicated, and should work closely with college of education advisors to be sure they have the most up-to-date guidance about the overall program expectations.

Accreditation

The preparation of teachers is professional preparation, and as such brings with it some features that are often unfamiliar to mathematics departments. Teachers in the nation's public schools must hold licenses; therefore, the programs that prepare them generally must be accredited, either through the state or through national organizations or both. Generally colleges of education bear the responsibility for maintaining accreditation. This involves preparing periodic self-study reports, addressing the standards and expectations of the accrediting agencies, organizing site visits for outside reviewers, and responding to reviewer concerns. The activities of subject matter departments fall within the purview of these accreditation agencies. Therefore, faculty in mathematics departments are called upon to help prepare self-study reports and to meet with accreditation teams. This means that someone in the department needs to be aware on a continuing basis of the accreditation issues and of changes and new trends that emerge.

The Need for Collaboration

Lack of mutual respect and cooperation between faculty in colleges of arts and sciences and faculty in education is a long-standing obstacle to the effective education of teachers.[5] Unfortunately, it is quite common for undergraduate students to hear faculty in mathematics criticize faculty in education for lacking high standards, for not understanding mathematics, or for teaching material that has no substance. And, conversely, students hear their education professors complain about poor teaching in the mathematics department or lack of attention by mathematics faculty to current issues such as the role of technology. A variety of programs, conferences, and initiatives that are intended to bring together administrators and faculty in colleges of arts and science with those in education have been initiated over the years, although there is little evidence that such programs have effect. At the level of specific mathematics departments, some things can help: hiring faculty members whose professional scholarship is in mathematics education; arranging joint or adjunct appointments for mathematics faculty in education; including faculty from education in programmatic review or development efforts; holding regular meetings of those who advise prospective secondary school teachers in mathematics and those who do so in education; arranging to host visiting teachers-in-residence from local high schools; and facilitating joint efforts on specialized projects in research, curriculum, or teacher education. Deans and chairs can enable such things to happen, but they will need to make special efforts to monitor and discourage the very negative conversations that sometimes happen along these lines.

Mathematics Education

The body of research about mathematics teaching and learning is substantial and growing. The most recent major synthesis (Grouws, 1992) includes a chapter on advanced mathematical thinking and a chapter on

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[5] Historically, it was in the university-based arenas that the liberal arts vs. professional education tension developed most strongly: "The collegiate [or university level] institutions entertained grave fears regarding the ability of the normal schools to properly equip teachers for the high schools, while the normal school was certain that the failure to provide practice teaching and the need to adapt subject matter to the mind of the high school student rendered collegiate education inadequate" (Pangburn, 1932, p. 52).

teacher education—both areas that bear upon aspects of the undergraduate major that contribute to teacher education.

The research on undergraduate mathematics education is rich in its documentation of student difficulties, in evidence about interventions that can support deep student understanding, and in portraying how technology can be used in the undergraduate arena to help support student learning (see, e.g., Dubinsky, Schoenfeld, & Kaput, 1994; Kaput, Schoenfeld, & Dubinsky, 1996; Schoenfeld, Kaput, & Dubinsky, 1998). Those who are concerned with the improvement of undergraduate education in general, and teacher education in particular, might find this literature useful. In fact, in most programs, prospective teachers read research about mathematics teaching and learning in their teacher education programs. Perhaps such reading lists would be good background material for the mathematics faculty who are providing their content background.

Generally, education research has little direct impact on practice, either at K–12 or in higher education. The impact of research tends to be indirect. In addition to the findings, both the theoretical perspectives and the methodologies of research can be useful. For instance, a standard research methodology used in gaining insights about a student's understanding of particular concepts is the clinical interview.[6] We have adapted this methodology and used it in mathematics courses for teachers, asking prospective teachers to interview a high school student on some difficult topic.

Relationship with the Major

An important consideration for mathematics departments is the reality that prospective secondary school teachers comprise a growing and significant fraction of the set of math majors in many departments. Although national data are not readily available, from reports at some institutions it seems plausible to guess that more than half of the mathematics majors nationally may intend to teach in secondary school. If this is the case, then mathematics departments must seriously consider what it takes to prepare a teacher mathematically. Will university departments be able to recognize that "teachers' mathematics" exists, is conceptually difficult, and should be offered through departments of mathematics? This is a non-trivial problem that deserves substantial intellectual and institutional resources.

Conclusion

At the core of departmental work relative to teacher education should be the very nature of the course and experiences for learning content that are provided to students. National conversations along these lines are moving very quickly to develop a concept of "mathematics for teaching," as described in previous sections. Exploration of what this would mean at the secondary school level is less well developed than at the elementary level, but if researchers begin to take up this line of work, there will ultimately be implications for, and challenges to, the traditional practice of having prospective teachers take only the mathematics courses taken by prospective graduate students. This is a big challenge, because the ideas are still nascent and the research is just taking shape. Little is known about what this involves in practice, let alone in teaching students how to do it. The issues may vary considerably by content area, and methods for helping prospective teachers learn about this are highly underdetermined. We hope that this paper will serve to broaden the community of mathematicians, mathematics educators, and mathematics education researchers who are willing to contribute to this important line of work.

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[6] In the interview the subject is presented with a mathematical problem and is asked to solve it and tell the interviewer what she/he is thinking along the way. The interviewer does some probing and prompting, but does not tutor the student or move the student toward a solution. The sole purpose is for the interviewer to elicit the student's thinking, so that a "theory" can be built about the student's understanding of the concept at hand. Usually clinical interviews are tape recorded, transcribed, coded and become part of a larger data set that can yield information about understanding of a particular concept across a range of students.

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