MAC-CPTM Situations Project

Situation 30: Translation of Functions

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Prompt

During a unit on functions, the translation of functions is discussed in a class. When the class encounters the function $y = (x - 2)^2 + 3$, one student notes that the vertical translation of 3 up makes sense when compared to the +3, but the horizontal translation of 2 to the right does not make sense with the –2 in the function.

Commentary

Note: Foci 1 and 2 use a parabola as the original function for convenience and because it was the function type specified in the prompt. One may translate any desired function similarly. Throughout this situation, the term *parent function* will be used to refer to the simplest example of any type of function (such as $f(x) = x^2$, $g(x) = x^3$, $h(x) = \cos(x)$), and the term *child function* will be used to refer to a transformation, in particular a translation, of a parent function (such as $f_1(x) = x^2 + 3$, $g_1(x) = (x - 1)^3$, $h_1(x) = \cos\left(x + \frac{\pi}{2}\right) - 4$).

Mathematical Foci

Mathematical Focus 1

Given a function $f(x)$, graphical representations allow easy comparison of the translations induced by various values of h and k in the transformed function $f(x - h) + k$.

A graphing utility can be used to examine the graphs of parent functions and child functions simultaneously. (see Figure 1). Looking at several iterations of the graph of $f(x - h) + k$ for different values of the parameters h and k, can allow one to see the relationship between the graph of the child function and the graph of the parent function relative to the parameter values.

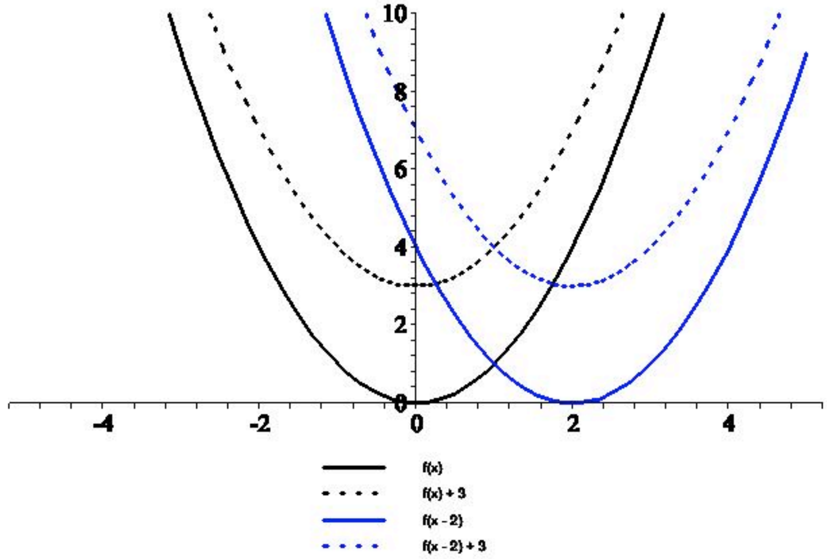


Figure 1

Mathematical Focus 2

A translation of a function $f(x)$ to the right occurs as a result of the composition $(f \circ g)(x)$ where $g(x) = x - h$ for positive h.

A numerical representation demonstrates that $(x-2)^2$ is 2 to the right of x^2 as a result of the output values of $x - 2$ being 2 less than the output values of x. This difference of 2 means that, in the table, $x - 2$ will produce the same output values as x, just 2 units earlier. Thus, $f(x-2)$ will have the same output values as $f(x)$, but 2 units later. This '2 units later' is along the x-axis, and is what yields a horizontal shift to the right by 2.

		Parent	**Child**
x	$x-2$	$f(x)$	$f(x-2)$
-5	-7	25	49
-4	-6	16	36
-3	-5	9	25
-2	-4	4	16
-1	-3	1	9
0	-2	0	4
1	-1	1	1
2	0	4	0
3	1	9	1
4	2	16	4
5	3	25	9

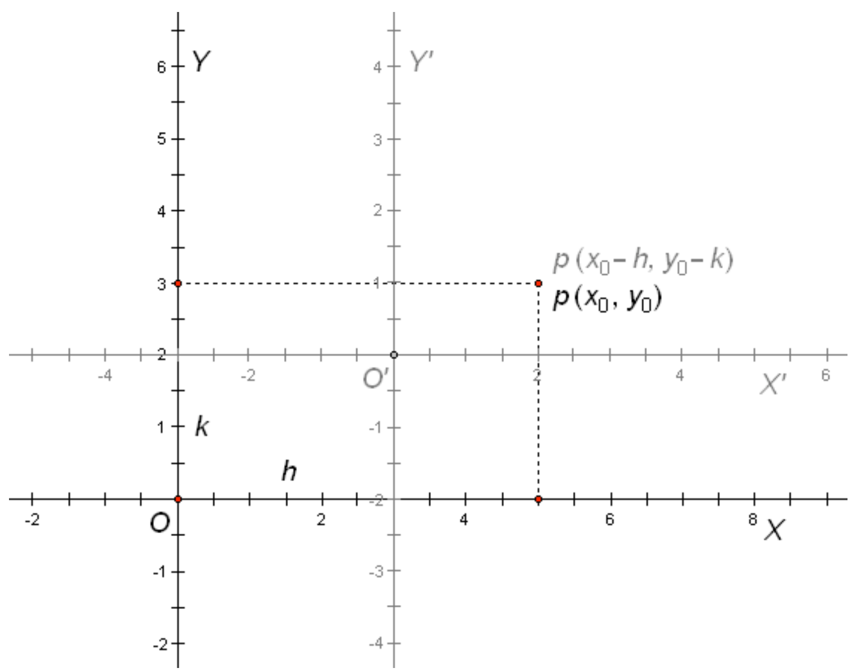
Figure 2

Mathematical Focus 3
The translation of a function (and its graph) can be thought of as the translation of the underlying coordinate axes.

Let $y = f(x)$ be a function graphed in relation to the axes, X and Y, and let $p = (x_0, y_0)$ be a point on that graph. Suppose there exists another set of axes X' and Y' (parallel to X and Y) whose origin has coordinates (h, k) in relation to X and Y. Then the coordinates of p in relation to the axes X' and Y' are $(x_0 - h, y_0 - k)$, as illustrated in Figure 3. Thus, the graph of the function f is of the form $y - k = f(x - h)$ in relation to the axes X' and Y', which can be written as $y = f(x - h) + k$.

For the parent function $y = x^2$, the child function $y = (x - 2)^2 + 3$ could be thought of as the formula for $y = x^2$ in relation to a new set of axes (parallel to the original) with origin at $(2, 3)$ as illustrated in Figure 4.

Figure 3



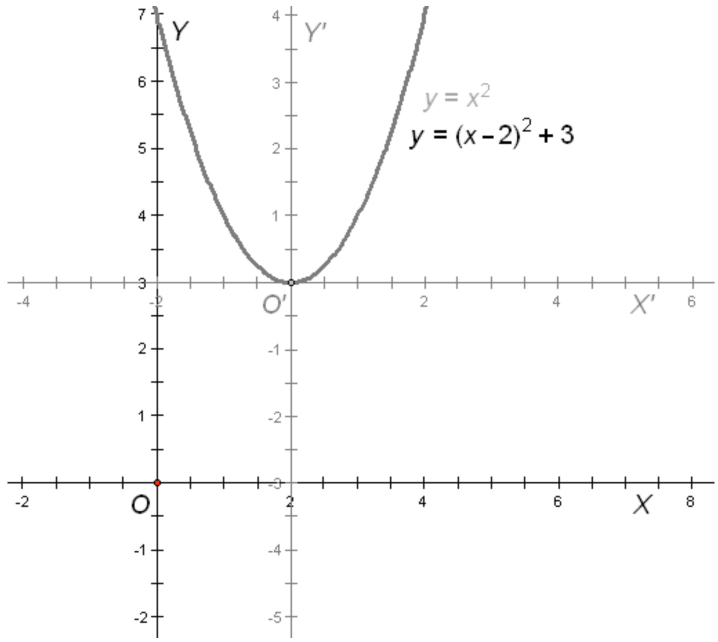


Figure 4

Post-Commentary

Mathematical Focus 3 draws on: Smail, L. L. (1953). *Analytic Geometry and Calculus*. Appleton-Century-Crofts, New York.