# Situation 04: Bull's Eye!
## Prepared at Penn State
## Mid-Atlantic Center for Mathematics Teaching and Learning
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## Prompt

During the lesson on the previous day, the class began to talk about mean, mode, median, and standard deviation. The students seemed to have learned something about mean, mode and median in an earlier grade.  However, they struggled with the symbols and procedures involved in standard deviation.  The purpose of this lesson was to revisit standard deviation. The teacher was quick to note that this was an advanced seventh grade class.

The teacher presented the formula and had students work through an example of computing standard deviation with data from a summer job context.  The following written work developed during the example:

| $x$ | $\bar{x}$ | $(x - \bar{x})$ | $(x - \bar{x})^2$ |
|-----|-----|-----|-----|
| 140 | 200 | -60 | 3600 |
| 190 | 200 | -10 | 100 |
| 210 | 200 | 10 | 100 |
| 260 | 200 | 60 | 3600 |

$$\sum (x_i - \bar{x})^2 = 7400$$

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum \left(x - \bar{x}\right)^2}{n}} \qquad \sigma = \sqrt{1850}$$

$$\sigma = \sqrt{\frac{7,400}{4}} \qquad \sigma \approx 43.01$$

The teacher then said, "Standard deviation is a measure of the consistency of our data set. Do you know what consistency means?"  To explain "consistency" the teacher used the idea of throwing darts.  One student pursued the analogy, "If you hit the bull's eye your standard deviation would be lower.  But if you're all over the board, your standard deviation would be higher."  The student drew the following picture to illustrate his idea:
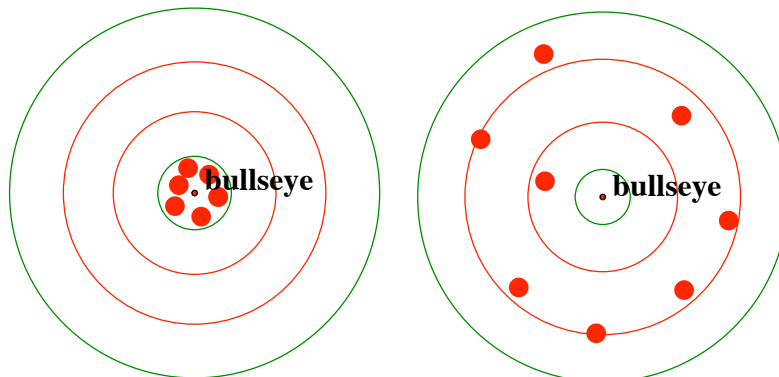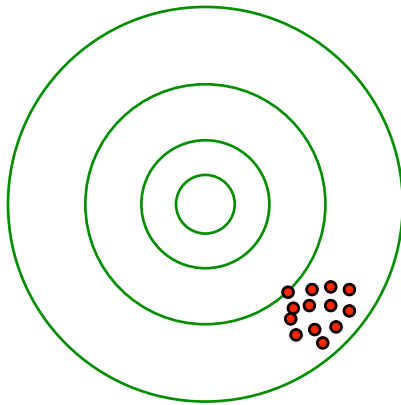
A student raised her hand and asked, "But what does this tell us about what we are trying to find?"
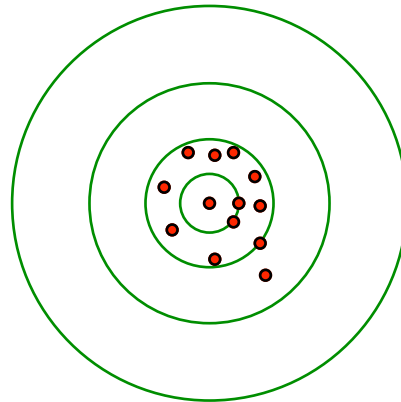
## Commentary

<See Mathematics used comments at the end of each focus>

## Mathematical Foci

### *Mathematical Focus 1*



**Target A**

**Target B**

Targets A and B above show different patterns of data. It is important not to confuse the center point of a target with the actual mean position of the data points. The diagram can be confusing if we don't keep in mind that it is an analogy, and as a representation does not capture the notion of mean as the mean position of the data points. If mean is interpreted as the mean position of the data points, then Target A will have the smaller standard deviation in spite of the fact that none of the hits were near the center of the target.

### Mathematics used for Mathematical Focus 1

- Understand that mean needs to be defined rather than assigned to some characteristic.
- Understand that the value of the mean depends on the data.

### *Mathematical Focus 2*

Provide students with a computer-generated sketch (GSP or Fathom) of hits on a circular target on which they can manipulate the data and which automatically updates the standard deviation. Have students experiment and generate

hypotheses about what makes the standard deviation smaller or larger. Have them generate justifications for their conjectures based on the standard deviation formula

## Mathematics used for Mathematical Path 2

- Understand that the value of the mean depends on the data.
- Understand the effects of changing data on a calculation that depends on the mean of the data points.
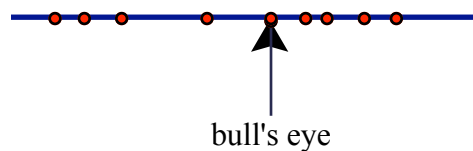
## *Mathematical Focus 3*

In the formula, we have $\sigma$, x, $\bar{x}$, and n. The number of darts is n. Each of places where a dart lands represents a value of x, or a data point. The bull's eye is used in lieu of the mean, $\bar{x}$. Each distance between a dart and the bull's eye is associated with a value of $(x - \bar{x})$, in absolute value terms.

## Mathematics used for Mathematical Path 3

- Conception of l$(x - \bar{x})$l as a distance
- Conception of $\bar{x}$ as a measure of central tendency
- Knowing distance is a non-negative measure but $(x - \bar{x})$ may have negative values
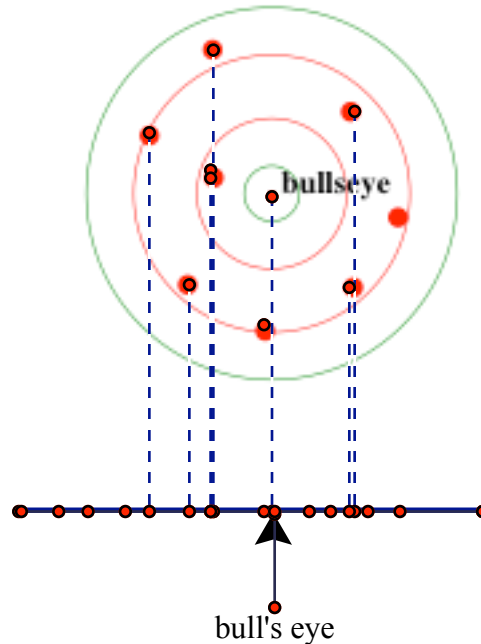
## *Mathematical Focus 4*

The number of darts is n. Each of places where a dart lands represents a value of x, or a data point. The bull's eye represents the mean, $\bar{x}$. Each distance between a dart and the bull's eye is a value of $(x - \bar{x})$, in absolute value terms. To allow for negative, zero, and positive values of $(x - \bar{x})$, we might think about a target whose shape is a line segment rather than a circle:

bull's eye

## Mathematics used for Mathematical Path 4

- Conception of l$(x - \bar{x})$l as a distance
- Conception of $\bar{x}$ as a measure of central tendency
- Knowing distance is a non-negative measure but $(x - \bar{x})$ may have negative values
- Knowing that positive/negative values could be implied in a one-dimensional representation

- Accuracy projecting 2D representation features to 1D representation, as in knowing that the projection implied in the following figure does not maintain the magnitudes of $(x - \bar{x})$:



bull's eye

## *Mathematical Focus 5*

The bull's eye diagram focuses on comparisons of points to the mean like the box-and-whisker plot focuses on the median as the "center" of the data set. The bull's eye diagram allows us to see something about the spread of the data. Since the standard deviation is calculated based on the mean rather than the median, we cannot use the box-and-whisker plot to "see" the standard deviation. The bull's eye diagram helps us to understand what different values for $\sigma$ tell us about the distribution of the data.

### Mathematics used for Mathematical Focus 5

- Understand mean and median as measures of central tendency and standard deviation as a measure of spread

# References

None