

## MAC-CPTM Situations Project

### Situation 04: Bull's Eye!

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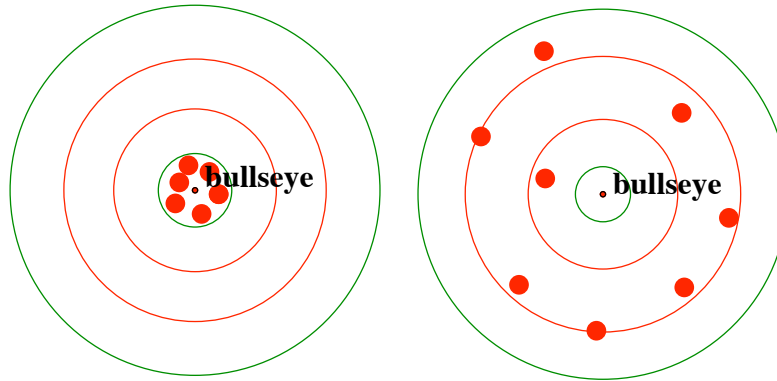
#### **Prompt**

In prior lessons, students learned to compute mean, mode and median. The teacher presented the formula for standard deviation and had students work through an example of computing standard deviation with data from a summer job context. The following written work developed during the example:

$x$	$\bar{x}$	$(x - \bar{x})$	$(x - \bar{x})^2$
140	200	-60	3600
190	200	-10	100
210	200	10	100
260	200	60	3600

$$\sum (x_i - \bar{x})^2 = 7400$$
$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$
$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} \quad \sigma = \sqrt{1850}$$
$$\sigma = \sqrt{\frac{7,400}{4}} \quad \sigma \approx 43.01$$

The teacher then said, “Standard deviation is a measure of the consistency of our data set. Do you know what consistency means?” To explain “consistency” the teacher used the idea of throwing darts. One student pursued the analogy, “If you hit the bull’s eye your standard deviation would be lower. But if you’re all over the board, your standard deviation would be higher.” The student drew the following picture to illustrate his idea:



A student raised her hand and asked, “But what does this tell us about what we are trying to find?”

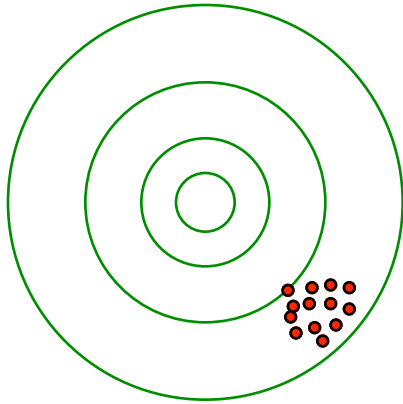
### **Commentary**

This prompt seems to deal with the analogy of the data points, mean, and the standard deviation of a data set as a dartboard target and the places on the target where the darts landed. We have to be explicit about what each part of the visual representation denotes. Also, it is important to know what each statistical term means so as to know how to represent it visually. All the foci look at various aspects of this visual representation of the statistical terms. With mathematical entities like mean and standard deviation that depend on each other, it is important that the representation capture the entities as well as their interrelationship.

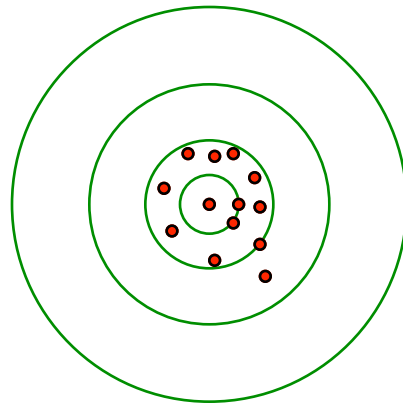
### **Mathematical Foci**

#### **Mathematical Focus 1**

*A robust representation of a data set and its mean and standard deviation allows for the mean and standard deviation to be defined by that data set and for the representation to reflect the relationship between mean and standard deviation.*



**Target A**



**Target B**

The two Targets A and B, shown in this Focus, show different patterns of data. In Target A, it is important not to confuse the center point of a target or the bull's eye with the actual mean position of any set of data points. The diagram can be confusing if we don't keep in mind that it is an analogy, and as a representation does not capture the notion of mean as the mean position of the data points. If mean is interpreted as the mean position of the data points and the standard deviation is interpreted as the "average" distance that each data point is from the mean, then Target A will have the smaller standard deviation in spite of the fact that none of the hits were near the center of the target. This is because the data points represented on Target B are more spread out than those of Target A.

### **Mathematical Focus 2**

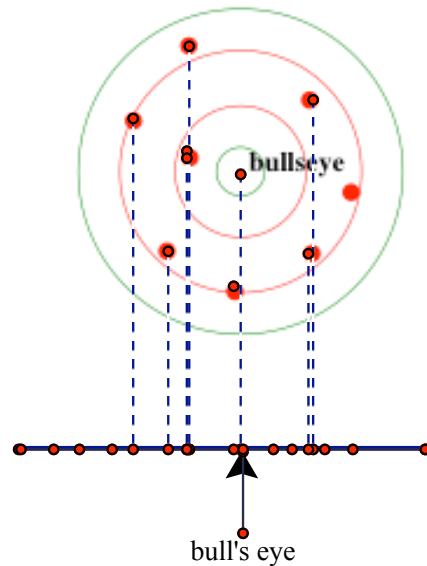
*If elements of a visual depiction of a formula represent symbolic elements of the formula, the relative positions of those elements should be consistent with the meaning of the symbols.*

In the formula, we have  $\sigma$ ,  $x$ ,  $\bar{x}$ , and  $n$ . The number of darts is  $n$  and in analogous to the number of data points in the data set. Every data point or each value of  $x$  is represented by the place a dart lands. For the two targets shown in the prompt, the bull's eye is used in lieu of the mean,  $\bar{x}$ . Each distance between a dart and the bull's eye is associated with a value of  $|x - \bar{x}|$ .

### **Mathematical Focus 3**

*Different representations might underscore different features of the mathematics they represent such as those that emphasize either magnitude or sign of numbers.*

Let the diagrammatical representations be as shown in the following diagram. To allow for negative, zero, and positive values of  $(x - \bar{x})$ , we might think about a target whose shape is a line segment rather than a circle, as shown below.



In this linear representation, all the data points are transposed onto a line, that is, a two-dimensional representation is transformed into a one-dimensional representation. So, the bull's eye is now represented on the line as the origin. Also, now every value of  $(x - \bar{x})$  depends on if the data point is to the left or right of the bull's eye. If the data point is to the left of the bull's eye, then  $(x - \bar{x})$  is negative and if it is to the right of the bull's eye, then  $(x - \bar{x})$  is positive. This transformation of the representation shows that while distance is always positive, the value of  $(x - \bar{x})$  can be either positive, negative, or zero (if  $x$  and  $\bar{x}$  lie on the same vertical line initially), however, it is important to remember that this transformation only preserves the sign of  $(x - \bar{x})$  and not the magnitude.

#### **Mathematical Focus 4**

*The mean and median are measures of central tendency and the standard deviation is a measure of spread; different representation may be more or less effective in representing these two different types of measures.*

The bull's eye diagram focuses on comparisons of points to the mean like the box-and-whisker plot focuses on the median as the "center" of the data set. The bull's eye diagram allows us to see something about the spread of the data. Since the standard deviation is calculated based on the mean rather than the median, we cannot use the box-and-whisker plot to "see" the standard deviation. The bull's eye diagram helps us to understand what different values for  $\sigma$  tell us about the distribution of the data.

## **Post commentary**

Aspects of a visual representation include the kinds of objects represented, the relative positions of those objectives, and a sense of which elements depend on which other elements in the construction of the representation. The robustness of the representation depends upon how well these three aspects collectively represent the key features of the mathematics being represented. These three aspects apply to familiar representations (e.g. box and whiskers plot) as well as novel representations (e.g. bull's eye).