MAC-CPTM Situations Project

Situation 07: Temperature Conversion

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Prompt

Setting:

High school first-year Algebra class

Task:

Students were given the task of coming up with a formula that would convert Celsius temperatures to Fahrenheit temperatures, given that in Celsius 0° is the temperature at which water freezes and 100° is the temperature at which water boils, and given that in Fahrenheit 32° is the temperature at which water freezes and 212° is the temperature at which water boils.

The rationale for the task is that if one encounters a relatively unfamiliar Celsius temperature, one could use this formula to convert to an equivalent, perhaps more familiar in the United States, Fahrenheit temperature (or vice versa).

Mathematical activity that occurred:

One student developed a formula based on reasoning about the known values from the two temperature scales.

"Since 0 and 100 are the two values I know on the Celsius scale and 32 and 212 are the ones I know on the Fahrenheit scale, I can plot the points (0, 100) and (32, 212). If I have two points I can find the equation of the line passing through those two points.

(0, 100) means that the y-intercept is 100. The change in y is (212-100) over the change in x, (32-0), so the slope is $\frac{112}{32}$. Since $\frac{112}{32} = \frac{7 * 16}{2 * 16}$, if I cancel the 16s the slope is $\frac{7}{2}$. So the formula is $y = \frac{7}{2}x + 100$."

Commentary

Mathematical Foci

Mathematical Focus 1

If one has an equation of a line, $y = \frac{7}{2}x + 100$, it represents a (linear) functional relationship between two quantities in which there is an independent variable and a dependent variable. What are the two quantities that are related by this equation? If *y* is the dependent variable, then what quantity does *y* represent (Celsius temperature or Fahrenheit temperature)? Similarly, if *x* is the independent variable, then what quantity does *x* represent?

Alternatively, focusing on rate of change would require interpretation of the meaning of the slope ($\frac{7}{2}$) of the line that is the graph of $y = \frac{7}{2}x + 100$. What is it that changes 7 for each change of 2 in something else? Or, for each increase of 2 in __, what is it that increases by 7?

Mathematics used for Mathematical Focus 1

- Concept of (univariate, linear) function: function as relationship between two variables (dependent variable and independent variable)
- Rate of change, independent variable, dependent variable

Mathematical Focus 2

If one has two temperature scales, is it possible to relate the values on one scale to the values on the other scale? If one has two scales with different zero points and different-sized units, is it possible to find a function that maps values from one scale to corresponding values on the other scale? (*See CAS-IM Module II (Heid & Zbiek, 2004) exploration of composition of functions via problems involving scale conversions*.) This can be thought of as composition of functions. The first function converts (dilates) the Celsius scale from a 100-point scale to a 180-point scale that is comparable to the Fahrenheit freezing point to boiling point distance. This requires multiplication by $\frac{9}{5}$. The second function adds 32° to the now-180-point scale to translate the new 180-point scale so the freezing points, 0 and 32, on the two scales to align. So the function *f* that converts Celsius temperatures to Fahrenheit temperatures is $f = h \circ g$, where $g(x) = \frac{9}{5}x$ and $h(x) = x + 32$.

Mathematics used for Mathematical Focus 2

- Concept of scale (zero point, unit, etc.)
- Transformations (translation, dilation)
- Composition of functions (composition of transformations).

Mathematical Focus 3

If the equation $y = \frac{7}{2}x + 100$ is thought of in terms of a linear function *f*, with rule $f(x) = \frac{7}{2}x + 100$, then focusing on the function's output values generated by certain input values or finding input values the produce certain output values might require one to interpret the meaning of particular values. (This is a variation of the approach in which one "checks whether the student's formula works.") For example, what is the value of *f*(0) and what should it represent? If the student's function rule (equation relating *y* and *x*) is correct, then *f*(0) should be 32. But from the student's equation, $f(0) = \frac{7}{2}(0) + 100$, so $f(0) = 100$. That would mean that freezing on the Celsius scale (0° C) would correspond to 100° F, a hot, summer day's temperature.

Or, given the Fahrenheit temperature on a pleasant day (e.g., 77° F), what Celsius temperature would correspond to it? (In $f(x) = \frac{7}{2}x + 100$, where would one insert the value of 77?) If one solves $f(x) = 77$ for *x*, then $77 = \frac{7}{2}x + 100$, or $-23 = \frac{7}{2}x$. This means that $x = \frac{-46}{7}$, meaning that the 77° F would correspond to about –6.5° C, a temperature well below freezing.

Or, what is the Fahrenheit temperature that corresponds to 26° C, a pleasant-day temperature? In $f(x) = \frac{7}{2}x + 100$, would *f*(26) produce the required value? $f(26) = \frac{7}{2}(26) + 100$, or $f(26) = 191$, a Fahrenheit temperature that is near boiling, not a pleasant-day temperature.

A near-freezing (near-zero) Celsius temperature should produce a Fahrenheit temperature near 32. For example, is *f*(2) near 32? $f(2) = \frac{7}{2}(2) + 100$, or $f(2) = 107$, a value quite far from 32.

Mathematics used for Mathematical Focus 3

- Given equation in *x* and *y*, consider *y* as a function of *x*.
- Given function input, find output; given output, find input.
- Reasonableness of function values or inputs.

Mathematical Focus 4

Considering defining characteristics and properties of (linear) functions might yield insight into the nature of the function that the equation, $y = \frac{7}{2}x + 100$, represents. What is the domain of this linear function? What is the range? Is the function one-to-one? Answering these questions requires one to know something about the relationship represented by the function. [Glen's question: Is this a viable additional path?]

Mathematics used for Mathematical Focus 4

- Defining characteristics and properties of linear functions (domain, range, one-to-oneness, …)

Mathematical Focus 5

Look at another point lying on the line containing (0, 100) and (32, 212). What does a point between (0, 100) and (32, 212) represent? One might think of points on the line containing (0, 100) and (32, 212) as being (freezing point, boiling point) pairs on various temperature scales. So the point (2, 107) would represent freezing and boiling points for *some other temperature scale* for which 2° was the freezing point and 107° was the boiling point, and therefore, clearly not one Celsius value and its corresponding Fahrenheit value. [This might be clarified by including a figure with a coordinate system on which the points (0, 100) and (32, 212) are plotted.]

Given that the Kelvin scale is another common scale, one might consider the Kelvin scale's freezing and boiling points and see whether the point (Kelvin freezing temperature, Kelvin boiling temperature) lies on the line represented by the student's equation, $y = \frac{7}{2}x + 100$. That pair for the Kelvin scale is (273.15, 373,15), a point that does NOT lie on $y = \frac{7}{2}x + 100$.

Mathematics used for Mathematical Focus 5

- Interpretation of values in an ordered pair (meaning of points on a line)
- Determining whether a particular point (ordered pair) satisfies an equation (lies on the graph of the equation).

References

Heid, M. K., & Zbiek, R. M. (2004). <**need citation for CAS-IM Module II**>

One URL for the Kelvin Temperature scale:
http://www.infoplease.com/ce6/sci/A0827335.html (accessed June 9, 2005)