Situation 2: Undefined Slope vs. Zero Slope

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EMAT 6500 class
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Prompt:

A teacher in a 9th grade Coordinate Algebra class is teaching a lesson reviewing the concepts of the slope of a line before beginning a new unit that uses coordinates to prove simple geometric theorems algebraically. While reviewing the different types of slope, a student raises her hand and asks, "What is the difference between zero slope and undefined slope?"

Commentary:

The set of mathematical foci below bring attention to the background knowledge that one must have in order to understand the mathematical concept of slope. Slope is a very broad topic in mathematics and can be represented in many different ways depending on the content area. With significance placed on rate of change, its definition and misunderstandings need to be understood by teachers so that students might gain conceptual understanding of the topic. Misunderstandings include concerns with formulas and understanding slope in contextual, real – life examples. The first focus highlights the history of when and where the concept of slope was developed; the next three foci emphasize the concept of slope within algebra, geometry, trigonometry, and calculus. The last focus underscores the importance of knowing the properties of dividing by zero to address the concept of undefined slope.

Mathematical Foci:

Focus 1: *It is essential for teachers of mathematics to know the historical development of the concepts they teach because it leads to a greater insight of the "why's" and "how's" of mathematics. This first focus highlights the history of when, where, and how the concept of slope originated.*

Rene` Descartes (March 31, 1596 – February 11, 1650), was a prominent and noteworthy French mathematician and philosopher. Descartes made numerous advances within the mathematics world, however, one of the most vital influences that he made in mathematics was inventing the Cartesian coordinate system that is used prominently in plane geometry and many algebra based curriculums taught in school today. According to the New World Encyclopedia, Descartes founded analytic geometry in his work titled La Gèomètrie, which was crucial to the development of calculus and analysis later solidified by Isaac Newton and Gottfried Wilhelm von Leibniz. Many writers dispute that Descartes was the first to develop the coordinate system and analytic geometry. According to Ann Gantert, author of the Amsco Geometry textbook, Pierre de Fermat (1601 – 1665) actually independently developed analytic geometry a few years before Descartes, however, Descartes was the first to publish his work. As stated by James Newman, "Fermat may have preceded Descartes in stating problems of maxima and minima; but Descartes went far past Fermat in the use of symbols, in "arithmetizing" analytic geometry by extending it to equations of higher degree. Further, Descartes fixed a point's position in the plane by assigning two numbers (now known as coordinates) by calculating the point's distance from two perpendicular lines (now known as the x and y – axis)." Therefore, analytic geometry and the discovery of the Cartesian plane are ultimately credited to Descartes' discoveries in mathematics. Since linear equations and the concept of slope are established within the eighth grade curriculum solely using proportional relationships within the coordinate plane, many mathematicians recognize Descartes as the person who invented the mathematical concept of slope along with the slope formula.

Focus 2: *Algebraic and Geometric Conception of Slope: The slope of a line is a measure defined by the ratio of vertical change to horizontal change or "rise over run".*

There are three basic linear equations that are presented in mathematics, the point – slope form, the slope – intercept form, and the standard form.

Point – slope form: $(y - y_1) = m(x - x_1)$, where m is the slope, and (x_1, y_1) is some given point on the line.

Slope – intercept form: $y = mx + b$, m is the slope, b is the y – intercept (the value of y when x = 0).

Standard form: $Ax + By = C$, where x and y are variables and A, B, and C are integers, and A is non-negative, and A, B, and C have no common factors other than 1. Standard form can be converted into slope – intercept form through a few manipulations:

$$Ax + By = C$$
$$By = -Ax + C$$
$$y = -\frac{A}{B}x + \frac{C}{B}$$

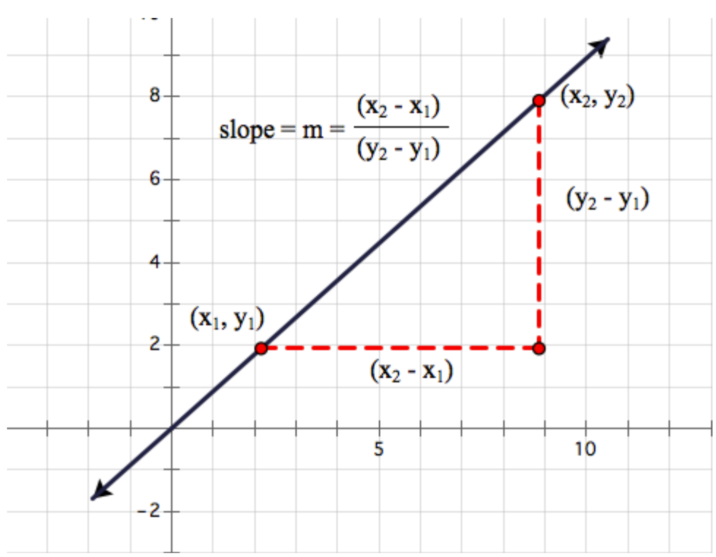
We now have converted the standard form equation into slope – intercept form where $-\frac{A}{B}$ is the slope and $\frac{C}{B}$ is the y – intercept. It is important as a teacher to know the relationship between the point – slope formula and the slope – intercept formula. Take a look at the relationship below:

Point slope formula: $(y - y_1) = m(x - x_1)$
Distribute the *m:* $(y - y_1) = mx - mx_1$
Add y_1 from both sides: $y = mx - mx_1 + y_1$
Commutative and Associate properties of addition: $y = mx + (y_1 - mx_1)$
Set b = $y_1 - mx_1$ to get $y = mx + b$

Traditional Algebra textbooks introduce the slope (or gradient) of a line as a measure defined by the ratio of vertical change to horizontal change, or more likely known as "rise over run". It is a rate of change in *y* with respect to *x* that measures the steepness of a line. In more mathematical terms, given a Cartesian plane, slope can be defined as change in the y – coordinates divided by the change in the x – coordinates. Thus, the slope of a line is defined based on the coordinates of two points.

Given two distinct points (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Although it doesn't matter which point you start with, being consistent is essential. For example, to calculate the slope you may use the formula $m = \frac{y_1 - y_2}{x_1 - x_2}$; however, you may not use the formula $m = \frac{y_1 - y_2}{x_2 - x_1}$. Whatever point you choose as the starting point in the numerator must be the same point you pick in the denominator (i.e. if you start with y_1 first in the numerator, you must start with x_1 in the denominator). The point slope formula can be derived from the slope formula above. Recall the slope formula as $m = \frac{y_2 - y_1}{x_2 - x_1}$. Say one of the points is a generic point (x, y) that can be anywhere on the line, and the other point is specific point say (x_1, y_1). Now, plug these points into the slope formula and rearrange the equation to obtain the point – slope formula.

$$m = \frac{y - y_1}{x - x_1}$$

$$(x - x_1)m = \frac{y - y_1}{x - x_1}(x - x_1)$$

$$(x - x_1)m = y - y_1$$

$$y - y_1 = m(x - x_1)$$

Thus, arriving at the point – slope formula. Knowing the relations between these formulas may help students grasp the concept of linear equations more thoroughly.

However, what if we are not given two distinct points (x_1, y_1) and (x_2, y_2)? What if $x_1 = x_2$ and $y_1 = y_2$? If we were to calculate the slope with $x_1 = x_2$ and $y_1 = y_2$, we would have zero divided by zero which is indeterminate.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{0}$$

Indeterminate is just a fancy word that essentially means undetermined, vague, or uncertain. You may ask why zero divided by zero is indeterminate? Well let's start taking numbers closer and closer to zero and divide them by themselves such as:

$$\frac{0.1}{0.1} = 1$$

$$\frac{0.001}{0.001} = 1$$

$$\frac{0.000000001}{0.000000001} = 1$$

Now, these can be positive or negative values, if we had $\frac{-0.1}{-0.1} = 1$, so why doesn't $\frac{0}{0} = 1$? Well, let's divide zero by numbers closer and closer to zero such as:

$$\frac{0}{0.1} = 0$$

$$\frac{0}{0.01} = 0$$

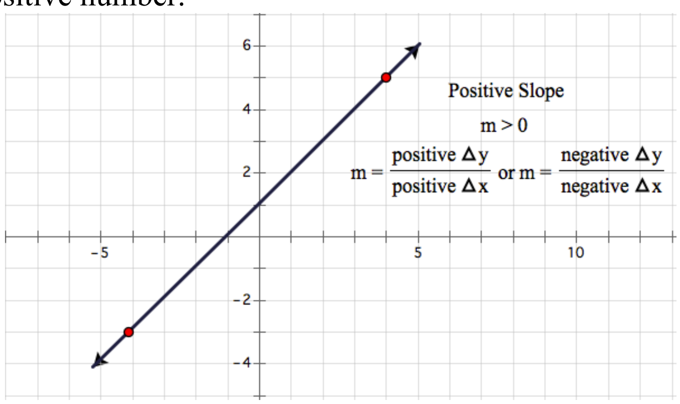
$$\frac{0}{0.00001} = 0$$

Thus, since both of these arguments are equally valid, mathematicians have concluded that zero divided by zero is indeterminate or undefined because the ratio zero divided zero is defined in terms of the limit as x approaches zero of the numerator divided by the limit as x approaches zero of the denominator. If the numerator approaches zero faster than the denominator approaches zero, the ratio is zero, but if the denominator approaches zero faster than the numerator approaches zero, the ratio approaches infinity. For a more simple explanation, consider $\frac{0}{0} = x$. If we solve this equation, we have 0 * x = 0; thus, the value of x can be any number.

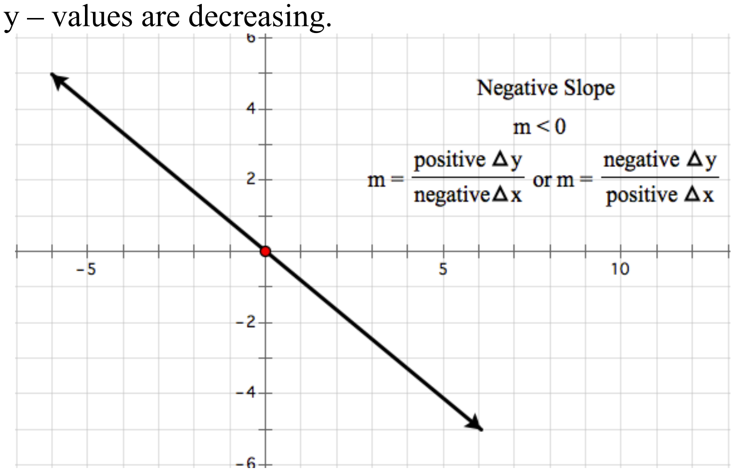
Many teachers may introduce the topic of slope by giving a real – life example such as ski slopes or the incline of a hill. The steeper and steeper a ski slope gets, the faster you are going to move down the hill. If one is walking up a hill, the slope is said to be positive; if one is walking down a hill, the slope is said to be negative; and if one is walking a flat line, the slope is said to be zero because there is no incline at all (i.e. no vertical change). However, what about a vertical line? Is one able to walk a vertical line that is straight up and down (i.e. perpendicular to a flat surface)? The slope of a vertical line is said to be undefined because there is an infinite incline, however, there is no horizontal change.

Slope can be positive, negative, zero, or undefined.

1) Positive slope means that the line is increasing from left to right or rising. Thus, as the x – values are increasing, the y – values are also increasing. A large positive number means the graph is steeper than for a smaller positive number.

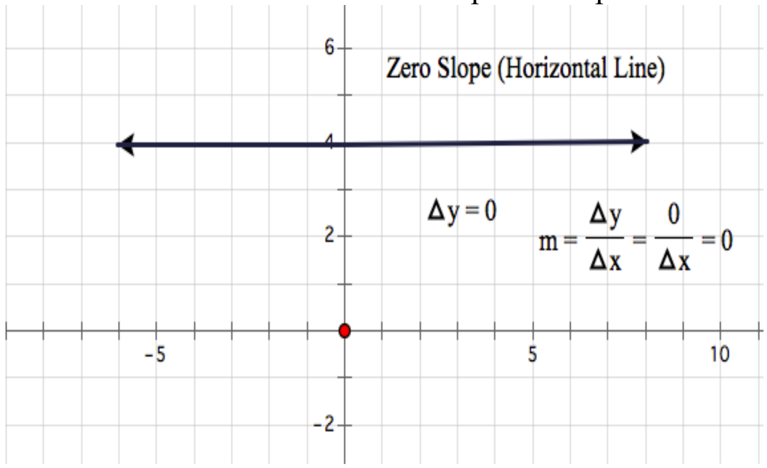


2) Negative slope means that the line is decreasing from right to left or falling. Thus, as the x – values are increasing, the y – values are decreasing.

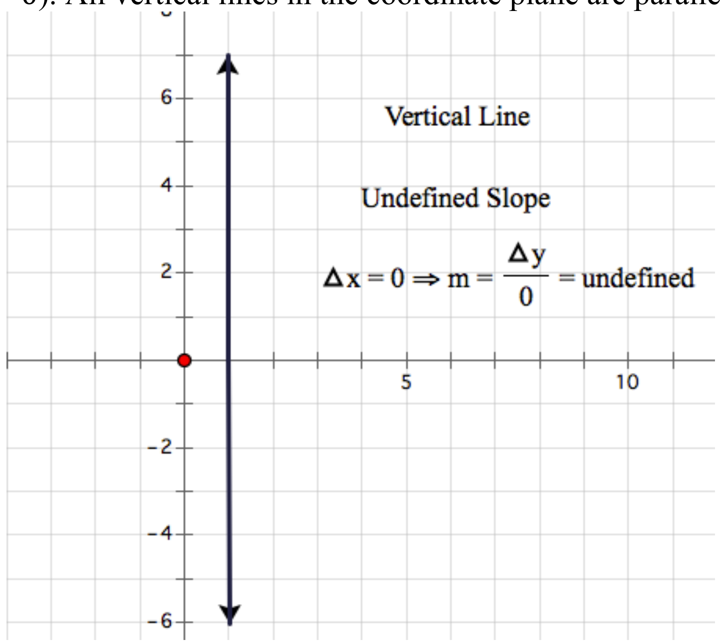


3) Zero slope means that the line is horizontal, thus there is no change in the y – values. The y – values stay constant on a horizontal line (i.e. rise = 0 because there is no vertical change, the quantity

$(y_2 - y_1) = 0$). All horizontal lines in the coordinate plane are parallel to the x – axis.



4) Undefined slope means that the line is vertical, thus there is no change in the x – values. The x – values stay constant on a vertical line (i.e. run = 0 because there is no horizontal change, the quantity $(x_2 - x_1) = 0$). All vertical lines in the coordinate plane are parallel to the y – axis.



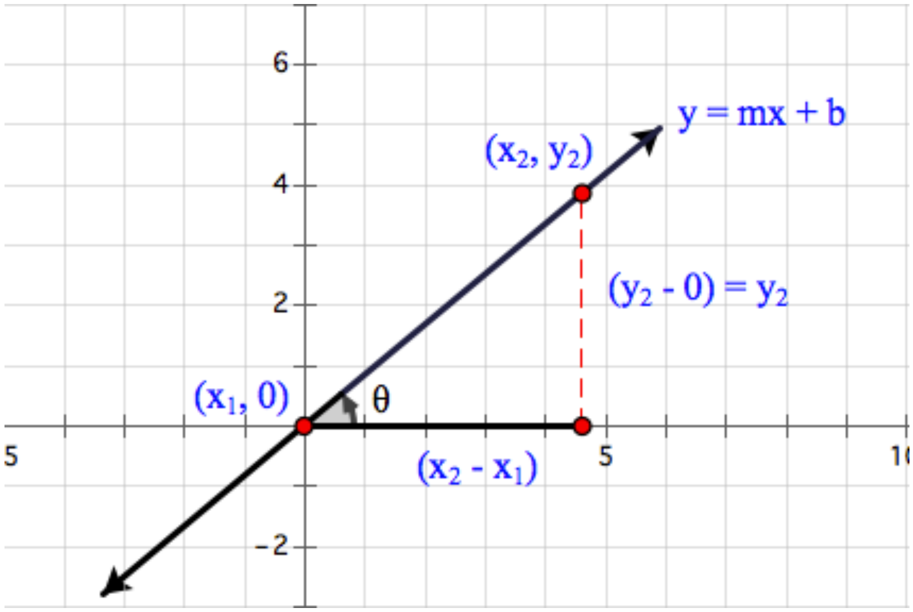
Focus 3: *Trigonometric Conception of Slope: Slope is the tangent of the angle made by a straight line with the x-axis.*

In trigonometry, the inclination of a non - horizontal line is the positive angle θ (less than π (i.e. the angle is always between $0°$ and $180°$)) measured counterclockwise from the *x*-axis to the line. The slope of a line y = mx + b is equal to the tangent of the angle of inclination (i.e. the angle it forms with the x – axis). The following proof is found in the 8th edition Pre – calculus textbook written by Ron Larson.

Theorem: If a non – vertical line has inclination θ and slope m, then $m = \tan\theta$.

Proof: If $m = 0$, the line is horizontal and $\theta = 0$. So, the result is true for horizontal lines because $m = 0$ and $\tan(0) = 0$. If *m* = undefined, the line is vertical and $\theta = 90°$. So, the result is true for vertical lines because *m* = undefined and $\tan(90°) = \frac{1}{0}$ = undefined.

If the line has a positive slope, it will intersect the x – axis at some point. Label this point $(x_1, 0)$, as shown in the figure below.



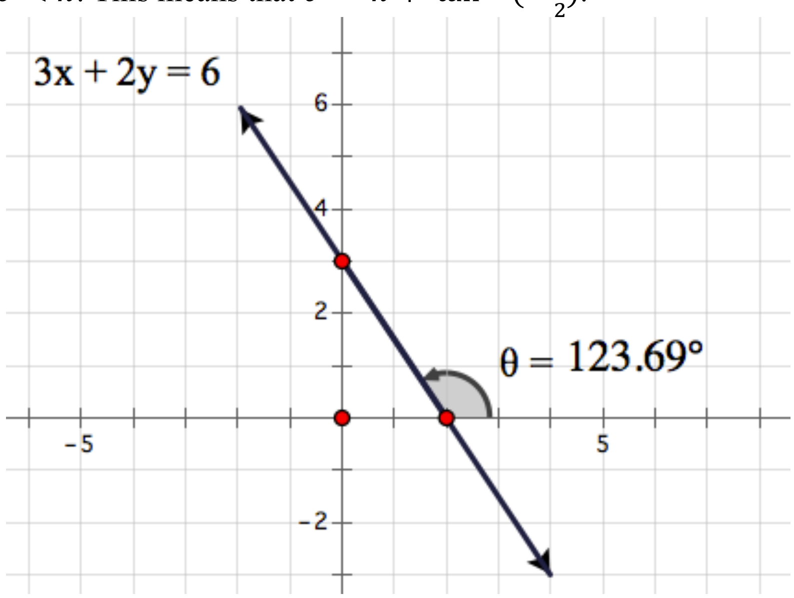
If (x_2, y_2) is a second point on the line $y = mx + b$, then the slope of the line is

$$m = \frac{y_2 - 0}{x_2 - x_1} = \frac{y_2}{x_2 - x_1} = \tan\theta$$

The case in which the line has a negative slope can be proved in a similar manner.

Example: Find the inclination of the line $3x + 2y = 6$

Solution: The slope of this line is $-\frac{3}{2}$, so it's inclination is determined by the equation $\tan(-\frac{3}{2})$. From the figure below, it follows that $\frac{\pi}{2} < \theta < \pi$. This means that $\theta = \pi + \tan^{-1}(-\frac{3}{2})$.



$$\theta = \pi + \tan^{-1}(-\frac{3}{2})$$

$$\theta \approx \pi + (-0.98279)$$

$$\theta \approx 2.158799$$

Thus, the angle of inclination is about 2.158799 radians or approximately 123.69 degrees.

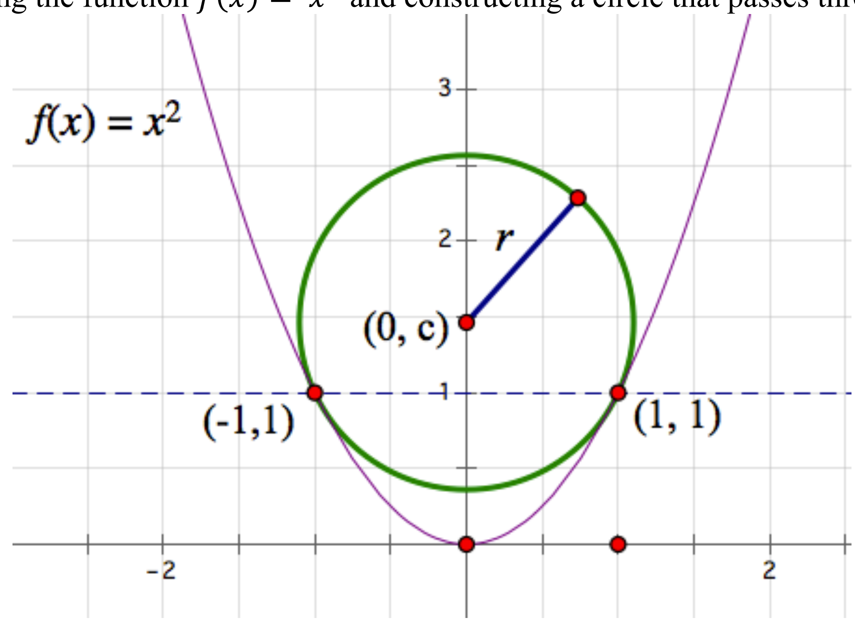
Focus 4: *Calculus Conception of Slope: Calculus typically begins with the study of derivatives and rates of change, using slopes of lines to develop these concepts*.

The problem of determining the slope of a curve fascinated many mathematicians for years. Archimedes made some of the first attempts where he was able to find the slope of spirals, but he did not have a general method for finding the slope of any curve. Descartes introduced a more general method for finding the slope of a curve using tangent circles. Descartes method of constructing the tangent of a line was based on the construction of a normal line, more commonly known as the line that is perpendicular to a curve at a particular point. He would begin with an arbitrary curve and a given point P. Then, he would find the equation of the circle that passes through the point P. His method ultimately started with the fact that the radius of a circle is always normal to the circle. Thus, his idea was essentially to find the equation of the circle that is tangent to the curve at a given point P, and use its radius to determine the tangent line.

The following example was developed by the Department of Mathematics and Statistics at Kennesaw State University:

Example 1: Find the slope of the tangent line to the function $f(x) = x^2$ through the point (1,1) using Descartes' method.

Begin by graphing the function $f(x) = x^2$ and constructing a circle that passes through the point (1,1).



Label the center of the circle (0, c) and the radius of the circle, *r*. Then, write the equation of the circle using the figure above.

$$x^2 + (y - c)^2 = r^2$$

Since the circle passes through the point (1, 1), we can set x = 1 and y = 1 in the above equation and simplify the equation to get a relation between *r* and *c*.

$$1^2 + (1 - c)^2 = r^2$$
$$1 + (1 - c - c + c^2) = r^2$$
$$1 + 1 - 2c + c^2 = r^2$$
$$r^2 = c^2 - 2c + 2$$

Then, substitute this *r* value into the original equation for the circle to get

$$x^2 + y^2 - 2yc + c^2 = c^2 - 2c + 2$$
$$x^2 + y^2 - 2yc + 2c - 2 = 0$$

Now, if we substitute $y = x^2$ into the above equation, we will find the y – values of the lying both on the circle and on the function $y = x^2$. Thus, we have the following quadratic:

$$y + y^2 - 2cy + 2c - 2 = 0$$

This quadratic will give at most two solutions; however, we are only interested in a single solution because we only want the curve to intersect the circle at a single point. Thus, there is only one solution to a quadratic equation when the determinant $(b^2 - 4ac)$ is equal to 0. Thus, we have

$$y^2 + (1 - 2c)y + (2c - 2) = 0$$
$$a = 1, b = (1 - 2c), c = (2c - 2)$$

Calculate the determinant:

$$(1 - 2c)^2 - \left(4 * 1 * (2c - 2)\right) = 0$$
$$(4c^2 - 4c + 1) - (8c - 8) = 0$$
$$4c^2 - 12c + 9 = 0$$
$$(2c - 3)^2 = 0$$

Solving this equation for *c* gives a value of $\frac{3}{2}$.

Now we have the center of the circle as $\left(0, \frac{3}{2}\right)$ and the point (1,1). Now, we calculate the slope of the line passing through the radius of the circle as

$$slope\ of\ radial\ line = \frac{\frac{3}{2} - 1}{0 - 1} = -\frac{1}{2}$$

Since the slope of the tangent line is perpendicular to the slope of the line through the radius of the circle, the slope of the tangent line will be opposite reciprocal of the slope of the radial line (i.e. the slope of the tangent line to the curve $y = x^2$ at the point (1,1) will be 2).

The more common notion of finding the tangent line to the graph of a nonlinear function is to use the limit approach. According to the textbook titled *Calculus* written by Ron Larson and Bruce H. Edwards, the definition of a tangent line with slope m is: "If *f* is defined on an open interval containing *c*, and if the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through the point (c, f(c)) with slope *m* is the tangent line to the graph of *f* at the point (c, f(c))" (page 97). The following examples are found in the *Calculus* textbook written by Ron Larson and Bruce H. Edwards:

Example: Find the slopes of the tangent lines to the graph of $f(x) = x^2 + 1$ at the points (0, 1) and (-1, 2) using the limit approach.

Let (c, f(c)) represent an arbitrary point on the graph of *f*. Then the slope of the tangent line at (c, f(c)) is given by

$$\lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = \lim_{\Delta x \to 0} \frac{[(c + \Delta x)^2 + 1] - (c^2 + 1)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{c^2 + 2c(\Delta x) + (\Delta x)^2 + 1 - c^2 + 1}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2c(\Delta x) + (\Delta x)^2}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2c + \Delta x}{\Delta x} = 2c$$

Therefore, the slope at any point (c, f(c)) on the graph of *f* is *m* = 2c. At the point (0,1), the slope is *m* = 2(0) = 0 and at the point (-1, 2), the slope is *m* = 2(-1) = - 2.

With the use of limits, we are able to differentiate functions. We can define the derivative of a function *f(x)* at a specified *x* – value *a* to be:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided this limit exists. There are three examples of the derivative occurring in real – life.

1) The slope of the tangent line to the curve *y = f(x),* at the point (*a, f(a))* is:

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

Example: Find the slope of the tangent line to the curve $y = \sqrt{2x+1}$ at the point (4, 3).

We can find the slope of the tangent line to the curve by computing the derivative at x = 4 as follows:

$$m = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4}$$

$$m = \lim_{x \to 4} \frac{\sqrt{2x+1} - \sqrt{2(4)+1}}{x - 4}$$

$$m = \lim_{x \to 4} \frac{\sqrt{2x+1} - 3}{x - 4}$$

Multiply the numerator and denominator by the conjugate

$$m = \lim_{x \to 4} \frac{\sqrt{2x+1} - 3}{x - 4} * \left(\frac{\sqrt{2x+1} + 3}{\sqrt{2x+1} + 3}\right)$$

$$m = \lim_{x \to 4} \frac{(2x+1) - 9}{(x-4)(\sqrt{2x+1} + 3)}$$

$$m = \lim_{x \to 4} \frac{2x - 8}{(x-4)(\sqrt{2x+1} + 3)}$$

$$m = \lim_{x \to 4} \frac{2(x - 4)}{(x-4)(\sqrt{2x+1} + 3)}$$

$$m = \lim_{x \to 4} \frac{2}{(\sqrt{2x+1} + 3)}$$

$$m = \frac{2}{\left(\sqrt{2(4)+1}+3\right)}$$

$$m = \frac{2}{(3+3)} = \frac{2}{6} = \frac{1}{3}$$

Thus, the slope of the tangent line to the curve $y = \sqrt{2x+1}$ at the point x = 4 is $m = \frac{1}{3}$. The slope of a tangent line can tell us a lot about how a curve is behaving. Horizontal tangent lines (i.e. tangent lines that have a slope equal to zero) are particularly important because that is where a minimum or maximum may occur, and it tells us when the graph is changing from increasing to decreasing or changing from decreasing to increasing. Let *f* be a differentiable function on the interval (a, b) then: If $f'(x)$ is greater than zero for *x* in (a, b), then *f* is increasing; If $f'(x)$ is less than zero for *x* in (a, b), then *f* is decreasing; and if $f'(x)$ is equal to zero for *x* in (a, b), then *f* is constant. The values of *x* such that the derivative is equal to zero or undefined are of special interest. We call *x* a critical number if either $f'(x) = 0$ or $f'(x) = undefined$.

2) The instantaneous velocity at time *t* = *a* of an object whose position is specified by *s(t)* is:

$$v = \lim_{h\to 0} \frac{s(a+h) - s(a)}{h} = \lim_{t\to a} \frac{s(t) - s(a)}{t-a} = s'(a)$$

To obtain the instantaneous velocity, we want to think of taking a small interval around the point of interest. If you want to find the instantaneous velocity or slope of a function, you would take the limit of the function around a certain point of interest (i.e. the derivative of the position function).

Example: Suppose that the position of a particle is given by the function $p = 1 + t^2 - t^3$, where *p* is in meters and *t* is in seconds. Find the instantaneous velocity at *t = 4 seconds* using the limit definition of the derivative.

First, since we are given the position of the particle, we must find the derivative of the position function in order to find the instantaneous velocity.

$$\lim_{h\to 0} \frac{1 + (t+h)^2 - (t+h)^3 - [1 + t^2 - t^3]}{h}$$

$$\lim_{h\to 0} \frac{1 + t^2 + 2th + h^2 - (t^3 + t^2h + 2t^2h + 2h^2t + h^2t + h^3) - [1 + t^2 - t^3]}{h}$$

$$\lim_{h\to 0} \frac{1 + t^2 + 2th + h^2 - (t^3 + 3t^2h + 3h^2t + h^3) - [1 + t^2 - t^3]}{h}$$

$$\lim_{h\to 0} \frac{1 + t^2 + 2th + h^2 - t^3 - 3t^2h - 3h^2t - h^3 - 1 - t^2 + t^3}{h}$$

$$\lim_{h\to 0} \frac{2th + h^2 - 3t^2h - 3h^2t - h^3}{h}$$

$$\lim_{h\to 0} \frac{h(2t + h - 3t^2 - 3ht - h^2)}{h}$$

Cancel the *h* in the numerator and denominator, then substitute *h* = 0 into the remaining expression to find the derivative.

$$2t + (0) - 3t^2 - 3(0)t - (0)^2 = 2t - 3t^2$$

$$p'(t) = v(t) = -3t^2 + 2t$$

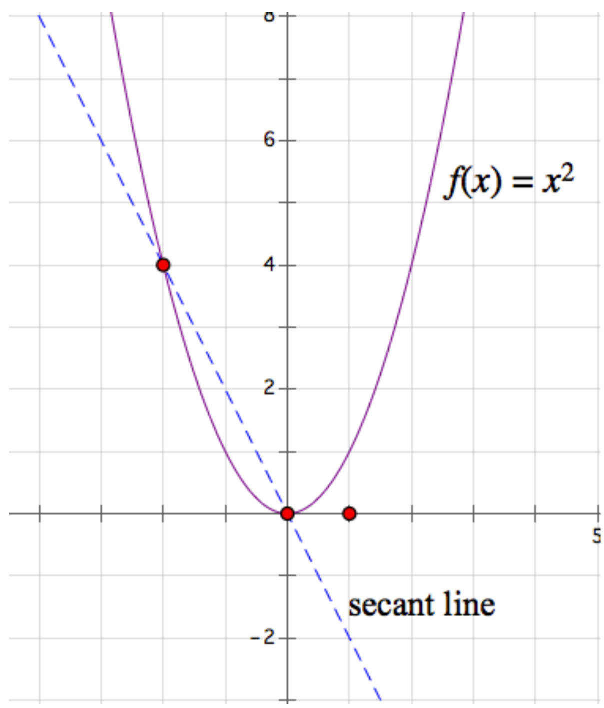
Find the instantaneous velocity at *t = 4 seconds* (i.e. Substitute t = 4 into the velocity function)

$$p'(t) = v(t) = -3(4)^2 + 2(4)$$
$$v(4) = -3(4)^2 + 2(4)$$
$$v(4) = -48 + 8 = -40\ meters/second$$

3) The instantaneous rate of change when $x = x_0$ of a quantity $Q(x)$ is:

$$r = \lim_{\Delta x \to 0} \frac{\Delta Q}{\Delta x} = \lim_{x_1 \to x_o} \frac{Q(x_1) - Q(x_0)}{x_1 - x_0} = Q'(x_0)$$

An important thing to remember is the difference between the *Average Rate of Change* and the *Instantaneous Rate of Change*. The *Average Rate of Change* is the slope of the secant line connecting two points along a curve (shown below). Thus, the slope of the secant line = $\frac{\Delta y}{\Delta x}$. The units on the *Average Rate of Change* are the units of dependent variable per one unit of the independent variable.



The *Instantaneous Rate of Change* (i.e. the rate of change at a specific point along a curve) is defined to be the limit of the average rates of change. Thus, the *Instantaneous Rate of Change* of a function *f* at a point *a,* is defined to be the limit of the average rates of change of *f* over shorter and shorter intervals around the point *a.* The following example was found the Wake Forest Department of Mathematics website:

Example: A company determines that the cost in dollars of manufacturing x units of a certain item is $C(x) = 100 + 5x - x^2$. Find the instantaneous rate of change of cost per item for manufacturing 1 item.

$$r = \lim_{x \to 1} \frac{C(x) - C(1)}{x - 1} = \lim_{x \to 1} \frac{100 + 5x - x^2 - [104]}{x - 1}$$

$$r = \lim_{x \to 1} \frac{-x^2 + 5x - 4}{x - 1}$$

$$r = \lim_{x \to 1} \frac{-(x-1)(x-4)}{x - 1}$$

$$r = \lim_{x \to 1} -(x - 4)$$
$$r = -(1 - 4) = 3\ dollars/item$$

Focus 5: *Division by Zero*

Division by zero has been a mathematical concept that has fascinated many teachers and students for many years. We touched on the concept of zero divided by zero being indeterminate in Focus 2; however, what if the numerator is any real number and the denominator is zero? Concrete arguments are usually used in elementary school to justify the impossibility of division by zero. A concrete argument to why division by zero is impossible is given below, and is then followed by a more formal proof found in abstract algebra.

Concrete Argument: A concrete argument uses a real – life example to express that division by zero is impossible.

$\frac{20}{2} = 10$ this means that if you had 20 blocks, you could separate them into ten groups of two.

$\frac{9}{3} = 3$ this means that if you had 9 blocks, you could separate them into three groups of three.

$\frac{4}{1} = 4$ this means that if you had 4 blocks, you could separate them into four groups of one.

$\frac{4}{0} =$? With this case you have to ask yourself, into how many groups of zero could you separate four blocks?

Obviously, it is impossible to separate four blocks into groups of zero, so dividing by zero is undefined.

Formal Argument: A more formal argument relies on the definition of division as the inverse of multiplication.

$\frac{1}{0}$ is said to be undefined because division is defined as the inverse of multiplication. Let $\frac{a}{b} = x$ is defined to mean that $b * x = a$. There is no *x* such that $0 * x = 1$, since $0 * x = 0$ for all *x*. Thus, $\frac{1}{0}$ is undefined

Formal Proof: This is a formal proof of division by zero using the definition of divisibility.

Let x ∈ ℤ. Prove that 0|x if and only if x = 0.

Solution: Since the theorem is an if-and-only-if statement, a proof must demonstrate the statement in both directions, i.e. you must prove both of the following statements:

1. If 0|x then x = 0.
2. If x = 0 then 0|x.

For the proof of statement #1, we assume that 0|x. By the definition of divisibility, there must be an integer n such that 0 · n = x. However, we know, from the properties of integer multiplication, that the product of 0 and any other integer is always zero. Hence 0·n = 0, and so x = 0.

For the proof of statement #2, we assume that x = 0. Properties of integer multiplication allows us to express 0 as zero times the integer 1 - that is, 0 = 0 · 1, for example. So now we have x = 0 = 0 · 1. The definition of divisibility allows us to conclude that 0|x.

For more information on the concept of *Division by Zero* please visit Dr. Jim Wilson's website: http://jwilson.coe.uga.edu/EMAT6500/EMAT6500.html and look at Situation 46!

Post Commentary:

From my experience with student teaching, I realized that my students were proficient in calculating the value of the slope (i.e. the number itself), but rather inexperienced in connecting meaning to the number. Thus, the conceptual understanding of what the slope actually means in a contextual setting is important for students to develop. In addition, many students are curious to why division by zero is impossible and leads to an undefined solution; thus, I think it is very important for teachers to know how to explain it in layman's terms that is easy for the students to understand. The mathematical concept of slope is also focused on within statistics and physics. In the Common Core Georgia Performance Standards, students are required to graph scatter plots and interpret the correlation coefficient, find the equation of the regression line, and interpret the slope and intercept of the regression line. In physics, students might have to determine the slope on a velocity – time graph or a position – time graph, along with many other applications.

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