COOPERATIVE LEARNING IN MATHEMATICS

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Connecting Research to Teaching

Roza Leikin and Orit Zaslavsky

COOPERATIVE LEARNING IN MATHEMATICS

> *How can the teacher best organize and manage the classroom during cooperative work so that discipline prone problems do not arise, interaction between students primarily involves task, and pupils still have sufficient freedom to contribute and to participate in the group discussion?*
>
> (Good, Mulryan, and McCaslin 1992, 185)

In this article we describe a method of implementing a cooperative-learning setting that we call *exchange of knowledge*. The design meets the goals suggested by Good, Mulryan, and McCaslin and gives students an opportunity to gain experience with some learning material and then to explain it to others. This method was developed on the basis of guidelines for cooperative learning in mathematics classrooms (Arhipova and Sokolov 1988). This setting was implemented and investigated for a variety of mathematics topics in secondary school with students of different age groups and ability levels in mathematics (Leikin 1993; Leikin and Zaslavsky 1997). We hope that our discussion of the exchange-of-knowledge method will furnish specific suggestions for promoting cooperative learning in your classroom, as well as a framework for considering the issues involved in evaluating cooperative-learning methods in general.

Students mutually and positively depend on one another

WHAT IS COOPERATIVE LEARNING?

Davidson (1990a) notes that it is difficult to precisely define *cooperative learning* because of the large variety of learning settings that are regarded as facilitating cooperative learning and the differences among them. However, on the basis of information in Artzt and Newman (1990) and Sutton (1992), we propose four necessary conditions that together constitute a cooperative-learning setting:

- Students learn in small groups with two to six members in a group.
- The learning tasks in which students are engaged require that the students mutually and positively depend on one another and on the group's work as a whole.
- The learning environment offers all members of the group an equal opportunity to interact with one another regarding the learning tasks and encourages them to communicate their ideas in various ways, for example, verbally.
- Each member of the group has a responsibility to contribute to the group work and is accountable for the learning progress of the group.

To be cooperative, a learning setting should ensure the existence of all these conditions. Contrary to common belief, forming groups in the classroom is not sufficient to create a genuine cooperative-learning setting. Of the four conditions, we consider the third to be particularly significant (Bishop 1985; Clement 1991; Jaworski 1992).

THE EXCHANGE-OF-KNOWLEDGE METHOD

We turn to a detailed description, based on Leikin (1993) and Leikin and Zaslavsky (1997), of the

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exchange-of-knowledge method. This learning method shares some characteristics with the jigsaw method (Aronson et al. 1978) in that it gives students an opportunity to play the role of a teacher and to offer explanations to their peers. However, the exchange-of-knowledge method also allows students to work individually when appropriate. In addition, the tasks are designed to have students work in pairs to ensure that every student has the opportunity to both study and teach each type of learning material.

The learning setting presented in this article resembles some features of Slavin's (1987) team-assisted individualization program, which fosters students' individual work within larger groups and encourages them to check and help each other when necessary by using given answer sheets. However, the proposed method develops more complex problem-solving and explaining activities. All students have to explain to one another mathematical ideas and principles, figure out for themselves how to solve problems, and decide on the acceptable or correct answers.

Description of the learning setting

The method is based on study cards and is carried out as follows:

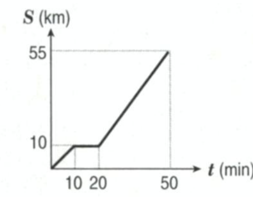
- Most of the time, students learn in pairs within a larger group of four students.
- Each student is required to explain to his or her partner how to solve the worked-out example in which the student has gained expertise on the previous card and to listen to the explanations given by the partner on how to deal with the worked-out example on a new card.
- Each student is required to solve a problem—similar to the previous worked-out example that the student's partner explained to the student—and is entitled, if needed, to ask the partner—who already tackled the problem—for help in solving it.
- After completing the work on a pair of cards, students change partners within the group. This move gives each group member an opportunity to act in the role of both a student and a teacher.

CARD 1

Relationship between a Function and Its Derivative Function

Part I—Example

Problem: The given graph represents the distance traveled by a car as a function of time. Draw a graph of speed of the car as a function of time.

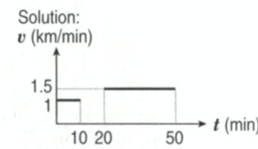


Explanation:
Speed is the rate of change of the distance over an interval of time.

An average speed is $v_a = \dfrac{\Delta S}{\Delta t}$.

Instantaneous speed is $v_i = \lim\limits_{\Delta t \to 0} \dfrac{\Delta S}{\Delta t} = S'$.

Instantaneous speed (rate of change of distance) is a slope of a graph of distance.



On the first segment: $\dfrac{\Delta S}{\Delta t} = \dfrac{10}{10} = 1$

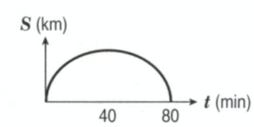
On the second segment: $\dfrac{\Delta S}{\Delta t} = \dfrac{0}{20 - 10} = 0$

On the third segment: $\dfrac{\Delta S}{\Delta t} = \dfrac{55 - 10}{50 - 20} = 1.5$

Part II—Solve a problem

Problem: The given graph represents the distance traveled by a car as a function of time.

Of the following graphs of functions, which could represent the speed of the car in this trip?



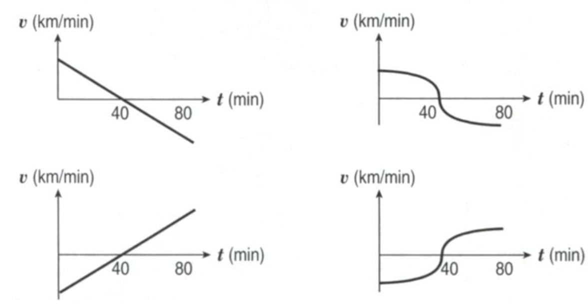


Fig. 1
Example of a working card (card 1)

Guidelines for preparing a set of study cards

Each set of cards constitutes a learning unit. Each set consists of two, four, or six study cards. The order in which the cards can be applied is not important. **Figures 1** and **2** show examples of learning cards.

Each card consists of two or three parts. Part 1 consists of a worked-out example. The extent of the explanations on the card depends on the students' level and on their learning experience in the topic. Part 2 includes a problem similar to the worked-out example on the first part of the card, for students' individual solutions. Part 3, if appropriate, includes an additional problem to be solved by more advanced students. For each study card a corresponding homework card is available.

Arrangement of learning within a classroom

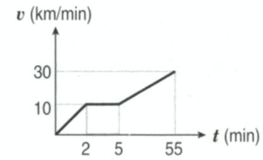
The learning setting is divided into two main stages, as shown in **figure 3:** groups of experts and groups for exchange of knowledge.

CARD 2

Relationship between a Function and Its Derivative Function

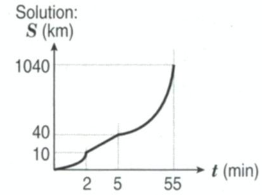
Part I—Example

Problem: The given graph represents the speed of a car as a function of time. Draw a graph of distance traveled by the car as a function of time.



Explanation: The distance traveled by the car in each segment of time is the area bounded between the graph of the speed and the x-axis on this segment of time.

Solution:



$S(2) = \frac{10 \cdot 2}{2} = 10$

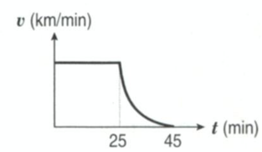
$S(5) = 10 + 10 \cdot (5 - 2) = 40$

$S(5) = 40 + \frac{(10 + 30) \cdot (55 - 5)}{2} = 40 + 1000 + 1040$

Part II—Solve a problem

Problem: The given graph represents the speed of a car as a function of time.

Of the following graphs of functions, which could represent distance traveled by the car in this trip?



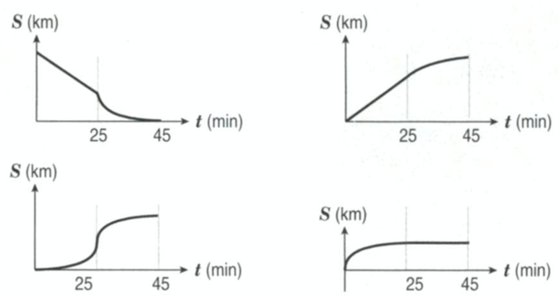


Fig. 2
Example of a working card (card 2)

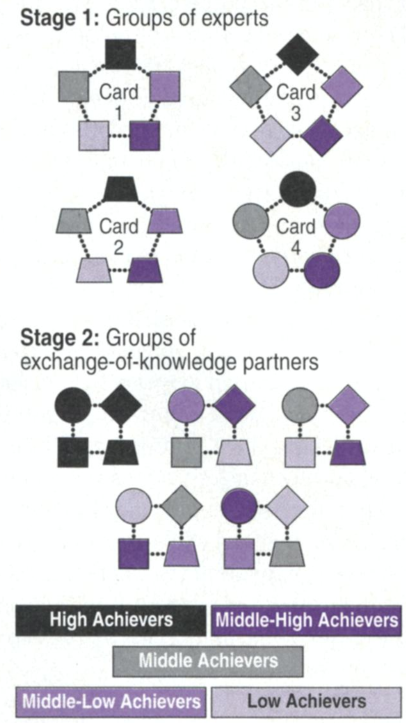


Fig. 3
Two stages of work

Groups of experts. All students learn within the groups of experts. No more than six students are in each group. Each student within a particular group gets the same card. The numbers of students receiving different cards are equal. All these groups contain students of varying achievement levels. The teacher makes sure that one student in each group is at the highest achievement level so that student can help the teacher check the pace and correctness of the group's work. The teacher monitors the work of this student, who is responsible for reviewing the work of all the group members. Students should understand the worked-out example presented in the first part of the card and are required to solve individually the problems given in the second part. Each student may ask for any needed help. Students compare their solutions within their groups and revise their solutions accordingly. The work in a group of experts is completed when the students agree on the solutions of the problems from part 2 of the card. The students then continue to work within new groups of exchange of knowledge.

Groups of exchange of knowledge. The number of students within a group should equal the number of cards within the learning unit. Each student has gained expertise in his or her own card, which is different from cards of the other students within the exchange-of-knowledge group, as in the jigsaw method. For example, if four cards are in the set of cards, the group contains four students, each of whom has a different card. High achievers learn within homogeneous groups, and students of middle and low levels work in heterogeneous groups to pace the work according to students' needs. This kind of arrangement enables the low achievers to feel more comfortable and believe that they can succeed in mathematics. The high achievers can learn additional material, as given on part 3 of the cards, for example. Middle-level students can develop confidence in their mathematical ability by helping other students. Within the exchange-of-knowledge groups, students work in pairs all the time.

Suppose that student Mike, who gained expertise in the learning material presented in card 1, c_1, is learning in the pair with student Nick, who gained expertise in card 2, c_2.

1. Mike explains to Nick part 1 of card 1 because he is working as a teacher explaining the worked-out example, asks Nick questions regarding the solution, verifies that Nick's understanding of the solution is acceptable, and answers Nick's questions.
2. Nick explains to Mike part 1 of card 2 in the same way.
3. When Mike and Nick finish their explanations, they have to solve part 2 of a new card simultaneously. They can ask each other questions and help each other if needed.
4. When they finish solving the problems from part 2 of the cards, Mike and Nick check each other's solutions and revise them.
5. If both of them accept their partners' solutions as correct, the work in the pair is completed.

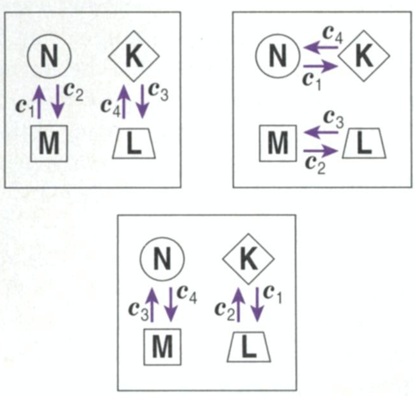


Fig. 4
Work within a group of exchange-of-knowledge partners

Pairs' work within a group for exchange of knowledge. (See **fig. 4.**)

1. Students Mike and Nick work as described in the previous section. At the same time, Kathy and Lora work with cards 3 and 4 (c_3, c_4). When the two pairs complete their work and each student has acquired expertise in two cards, they begin their next stage and move to work within a new pair.

2. Mike works with Lora, and Nick works with Kathy, using the cards that they received at the previous stage. When the students complete their work in these pairs, they return to their previous peer with the new card.

3. Mike works with Nick, and Lora works with Kathy, using the cards that they received at the previous stage. When students complete their work in these pairs, the unit has been completed.

In this way, by the end of the last stage, students have worked with all the learning cards, learned from a worked-out example of one of the problems, received from their partners explanations regarding the other three tasks, explained worked-out examples of three of the four cards in the learning unit, and solved all types of problems individually.

This cooperative setting requires that students know the underlying principles and have factual knowledge relevant to solving the unit problems.

If the number of students is not evenly divisible into groups, a teacher can let a student who is at a low achievement level work with a middle-level achiever all the time. This pair then works as one student in the learning arrangement. They solve problems individually and explain different cards to their partners in turn.

LEARNING OUTCOMES

In Leikin and Zaslavsky's study (1997), students' learning in traditional settings was compared with their learning by the exchange-of-knowledge method. Four middle-level ninth-grade classes were included in the study. This study investigated three main questions with respect to the experimental exchange-of-knowledge learning setting:

1. What is the effect of this cooperative small-group learning setting on students' activeness?
2. What kinds of students' interactions take place, and in particular, what kinds of help do students receive in this learning setting?
3. What are students' attitudes toward the experimental method?

The findings of this study show that the experimental small-group cooperative-learning setting facilitates a higher level of learning activities. Classroom observations indicated an increase in students' activeness. Altogether, students spent much more time actively involved in the experimental cooperative setting. We attributed this change to the increase in mathematical communications, which were defined in general as student–student and student–teacher interactions related to the learning material.

Mathematical communications increased

Observations pointing to these communicative interactions took the form of giving an explanation and posing a question or requesting help. These two types of communicative interactions, which we call *mathematical communication,* fall into what Webb (1991) calls students' verbal interactions. These two categories of mathematical communication are considered very active and desirable. The importance of mathematical communication is also manifested in the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989). We found that student-teacher learning interactions dominate whole-class settings, whereas student-student learning interactions tend to dominate the cooperative-learning setting. We suggest, therefore, that the increase of students' activeness in the experimental classroom situations occurred mainly because of the opportunity for increased student-student learning interactions.

Students were asked to report about the types of help they received while solving individually a problem from part 2 of a learning card. The students' self-reports dealt with the part of the student-student learning interactions related to help. The high percent of instances in which students stated that they had requested help from their peers indicates that the supportive atmosphere created by the experimental-learning environment encourages students to ask for help despite their normal reluctance to do so (Newman and Goldin 1990). In fact, students received more help than they requested. This supportive atmosphere could be attributed to the special arrangement of the small groups.

We also examined students' attitudes with respect to the cooperative-learning setting. An extremely high percent of students, 90 percent, expressed positive attitudes toward the opportunities to pose questions and to explain the learning material to their peers. Students' overall attitudes toward the learning method were highly positive.

Ninety percent expressed positive attitudes

What types of help did students offer each one another in the experimental small-group cooperative-learning setting? Explanations were the predominant type of help. According to Webb, this type of help is the most powerful. "The content-related help that students give each other in small groups might be considered to lie on the continuum according to amount of elaboration. Detailed explanations would be at the high end of such an elaboration scale, merely stating the answer to a problem or exercise would be at the low end, and providing other kinds of information would fall in between the two extremes" (Webb 1991, 367). For most of the situations in which help was requested, the help offered included explanations. This finding indicates that the experimental cooperative-learning method allows students to construct explanations regarding underlying principles for solving mathematical problems.

GUIDELINES FOR FACILITATING COOPERATIVE LEARNING IN MATHEMATICS

Teachers can use many methods to facilitate cooperative learning. In designing a cooperative-learning setting in mathematics, special attention is usually given to the following issues (Hertz-Lazarowitz and Fuks 1987; Kroll, Masingila, and Mau 1992): (1) the structure of the cooperative groups, (2) students' interactions in each group, (3) interactions among the different groups, (4) learning tasks and the teacher's role in the classroom, and (5) assessment and evaluation of the learning process. These five criteria influence the type of cooperative-learning setting that takes place in the classroom and its success.

The structure of the cooperative group

The structure of a cooperative group is defined by the number of students within a group and by the degree of heterogeneity of a group.

Number of students within a cooperative group. The majority of the authors discussing cooperative learning refer to this issue (Artzt and Newman 1990; Davidson 1990a; Davidson 1990b; Slavin 1985; Webb 1985; Hertz-Lazarowitz and Fuks 1987). The number of the students in a group depends on the type of the mathematical activity that is intended to take place in the classroom. In general, four is the optimal number of members in a cooperative group. Some researchers recommend that students work in pairs and emphasize that working in pairs facilitates active learning. Others suggest that a group of six students is the best group size for a cooperative-learning setting. However, all the researchers agree that the number of students in a group should not exceed seven. The exchange-of-knowledge learning setting in this article gives students an opportunity to work in pairs within a larger group of four or six students.

Heterogeneity of a cooperative group. According to Davidson (1990b), heterogeneity of a small group is one of the most important issues when planning a cooperative-learning setting. Students learn better in groups of different ability levels, that is, heterogeneous groups (Davidson 1990a; Davidson 1990b; Slavin 1985). Note that students with high ability levels prefer to learn with students having similar ability levels. At the same time, students who have learning difficulties prefer to cooperate with students who are able to help them while learning. In the exchange-of-knowledge method, the heterogeneity of the small groups varies according to the different stages of learning. Students begin their learning in heterogeneous small groups with respect to students' achievement levels, and then high achievers continue their work in homogeneous groups.

Students' interactions in each group

One of the main purposes of cooperative-learning settings is to promote task-related interactions by students. The learning method facilitates students' interactions, for example, when students are required to switch roles. Students' interactions can also be enhanced by the nature of the task, for example, the specific task can call for an exchange of ideas. The types of interactions depend on the types of learning objectives (Sharan et al. 1980). The learning objectives can be determined for each student individually. Cooperation within the group then is mainly a means for achieving the objectives. However, the learning objective can be determined for the group as a whole, in which case cooperation is a necessary condition in the learning setting. In the exchange-of-knowledge setting, students are assigned both individual and group learning objectives.

Interactions among different groups

Interactions among the learning groups may or may not take place (Sharan et al. 1980). Students may present the results of their group work to the other groups, or they may finish their group work within the small group without communicating with members of the other groups. Interactions between various groups can be facilitated by some sort of competition. In the exchange-of-knowledge method, students switch from one working group to another on an individual basis and no interactions among the groups take place in the classroom.

Types of learning tasks and the teacher's role in the classroom

One crucial point of any cooperative-learning setting is the teacher's role in the classroom. The way in which the learning material is presented to the students and the way in which a teacher communicates with students during the group work influence students' learning interactions. In the exchange-of-knowledge setting, the teacher's role is to help students solve problems when they request help. The learning tasks are presented as worked-out examples. This design is intended to focus students' interactions on understanding these examples, by explaining to each other what they already know and by solving new problems similar to the worked-out examples.

Assessment and evaluation of the learning process

The teacher's ability to assess learning progress can influence the success of the learning process. The type of assessment depends on the type of learning objectives and setting. Individual objectives demand individual means of assessment, whereas group objectives imply group assessment. In the exchange-of-knowledge setting, the teacher assesses individual students' learning progress. The group does not have a goal of its own.

CONCLUSIONS

Cooperative-learning settings address many of the concerns that teachers have and give them ways to deal with some problems that they face in their classrooms. Moreover, educational research points out the great contribution of cooperative learning to academic and social fields of the learning process (Artzt and Newman 1990; Davidson 1990a; Davidson 1990b; Davidson and Kroll 1991; Kroll, Masingila, and Mau 1992; Slavin 1985; Weissglass 1990).

Some studies suggest that students with different levels of ability become more involved in task-related interactions as a result of cooperative learning and that students' attitudes toward school and toward the discipline become more positive. While learning mathematics in certain cooperative-learning settings, students often improve their problem-solving abilities, solve more abstract mathematical problems, and develop their mathematical understanding. With respect to mathematics achievement, some studies show that students' achievements do not change as a result of learning in a cooperative-learning environment, whereas other studies give empirical evidence that cooperative learning may improve students' mathematical achievements.

Explanations were the predominant type of help

Overall the main findings of our investigation are as follows:

1. The implementation of the exchange-of-knowledge settings promoted students' active explorations in the mathematics classroom.
2. A close examination of the nature of students' activities indicated an increase in students' mathematical communications.
3. An investigation of the types of help that students received while learning showed that verbal explanation is the predominant type of help received by the students.
4. Students' attitudes towards the exchange-of-knowledge method were positive.
5. Students' achievements in the experimental method were at least as good as those of students learning in the conventional way.

The exchange-of-knowledge method discussed in this article is an example of a structured setting that facilitates students' cooperative learning of mathematics. Several ways exist to design the learning of mathematics to promote students' activeness and communication. In this article we described general guidelines and principles that are helpful in planning mathematics lessons, so that teachers can adapt and implement them according to their inclinations and preferences. Not all classroom conditions equally lend themselves to such a learning setting. However, many of the ingredients discussed in the exchange-of-knowledge setting can make significant contributions to the mathematics classroom through facilitating students' mathematical communications. Mathematical communications can play an important role in learning mathematics. When communicating mathematically, students—

- enhance their understanding of mathematics,
- establish shared understanding of mathematics,

- become more active learners,
- learn in a comfortable environment, and
- assist the teacher in gaining insight into their thinking.

Thus, we recommend the exchange-of-knowledge method for implementation in mathematics lessons in secondary school.

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