



NATIONAL COUNCIL OF  
TEACHERS OF MATHEMATICS

---

International Perspectives: ACTIVE Learning of Mathematics

Author(s): JIM SMITH

Source: *Mathematics Teaching in the Middle School*, Vol. 5, No. 2 (OCTOBER 1999), pp. 108-110

Published by: [National Council of Teachers of Mathematics](#)

Stable URL: <http://www.jstor.org/stable/41180748>

Accessed: 02/10/2013 00:04

---

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at  
<http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



National Council of Teachers of Mathematics is collaborating with JSTOR to digitize, preserve and extend access to *Mathematics Teaching in the Middle School*.

<http://www.jstor.org>



International  
*Perspectives*

# ACTIVE

## Learning of Mathematics

JIM SMITH

**S**INCE THE COCKCROFT REPORT IN 1982, THERE HAS BEEN AN increasing emphasis on the use of active learning in school mathematics, with a typical and influential view being propounded in *Better Mathematics*: "Mathematics can be effectively learned only by involving pupils in experimenting, questioning, reflecting, discovering, inventing and discussing. Mathematics should be a kind of learning which requires a minimum of factual knowledge and a great deal of experience in dealing with situations using particular kinds of thinking skills" (Ahmed 1987, 24).

During the same period of time, there has been an increased explicit and implicit use of a constructivist epistemology, for example in an implicit way: "The teachers job is to organize and provide the sorts of experience which enable pupils to construct and develop their own understanding of mathematics, rather than simply communicate the ways in which they themselves understand the subject" (NCC 1989, para. 2.2).

And in an explicit way: "Many writers embed their view of active learning in a framework concerned with the nature of the intellectual activity taking place, most often located within a constructivist model of mathematical learning" (Kyriacou 1992, 312).

We can, with some consistency, summarize this particular constructivist framework as the hypothesis that human knowledge is personally constructed and consists of conjecture, unfalsified theories, modified theories and expectations.

A teacher working with this view of how we come to know would have to provide pupils with appropriate chances to create their own theories, to engage their mental model-making processes and to allow them the opportunity to develop ex-

---

JIM SMITH works at Sheffield Hallam University, Sheffield S10 2NA, England.

---

*"International Perspectives" examines mathematics topics that have been published and discussed in countries around the world. Readers who wish to share information from international sources should contact this department at NCTM, 1906 Association Drive, Reston, VA 20191-1593.*

---

*"Active Learning of Mathematics" by Jim Smith, which appeared in the March 1996 issue of Mathematics Teaching, is reprinted by permission of the Association of Teachers of Mathematics, Derby, DE23 8Y; all rights reserved.*

expectations in order to subject their theories to the test of reality. This [approach] strongly suggests that pupils must be actively engaged in constructing their understanding, and that the activities themselves must be judged mainly by their contribution in assisting pupils to construct their own understanding of concepts selected by the teacher.

Within this particular constructivist framework, learning tasks should be chosen with the specific intention of actively involving learners in seeking to understand their external world by creating and testing their own models of what is going on “out there” in reality. The teacher has a clear initial role in selecting activities that are expected to focus the attention of the learner on constructing the intended learning outcomes of the session. The mathematical activities should therefore be selected or designed to encourage the learner to link between external world and internal thought. This involves a consideration of presentation, pupil activity, reflection and socialization. Activity by itself is not enough.

## Presentation

WE MUST AIM TO PRESENT ANY MATHEMATICAL ACTIVITY in a way that invites pupils to fully engage their higher mental capacities. This can be by the use of a game, a puzzle, a surprise or some other intriguing challenge. In creating a challenge for learners, we must be aware of the need to choose an appropriate level of challenge; one that learners can perceive as offering them a realistic, but not certain, chance of meeting. The challenge can come from the teacher or from the pupil themselves. It can be in the form of a puzzle, a target to reach, a goal or a conflict to resolve.

An example of a presentation that involves all four of these aspects is to motivate pupils’ work on geometrical construction. One way in which I have done this is to show pupils a hexaflexagon, on the front of which is drawn a pattern and on the reverse, a different pattern. Pupils are asked to memorize the patterns, and whilst they attempt to do this, I flex the hexaflexagon whilst maintaining the same pattern on the front, then ask, “Who can remember the pattern on the back?” and surprise the class by showing that the pattern has “disappeared.” The goal is then set for the class members to make their own hexaflexagon, resolving the puzzling disappearance of the lost pattern. I have found this to be a simple yet highly motivating presentation, which sometimes generates a round of applause—I wish I could say that more often!

In creating the initial challenge, surprise and cognitive conflict can be useful to generate interest. For example, by asking pupils to work in pairs and giving one of each pair a basic calculator and the other a scientific calculator, we can create cognitive conflict and surprise if we ask pupils to carry out calculations (like  $2 + 3 \times 5$ ) that give different answers on the two calculators. The role of the presentation is to indicate to the learner the need for new or revised theories; the role of the challenge, surprise or cognitive conflict is to engage the learners’ full attention.

## Pupil Activity

TO SOME EXTENT THE ROLE OF PUPIL ACTIVITY IS clear; it is for learners to undertake in order to meet the challenge set by themselves, the teacher or the text. They should be trying to make sense of the challenge, the activity and their findings. It can therefore be to move toward the goal, or to attempt to resolve a cognitive conflict, or even to explore the extent of the cognitive conflict. For example, to continue with the calculator challenge, we might ask pupils to find out as many sums as they can that lead to different answers on the different calculators.

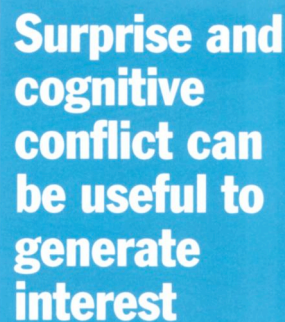
The key aspect of any learning activity is that it must be constructed or chosen to demand mental involvement; perhaps this is best achieved by requiring pupils to deal with new, unfamiliar and nonroutine activity. Familiar mathematics can often be packaged in an unfamiliar way; for example, I recently worked with a Y10 top set class studying graphical inequalities. To do this, I began by placing the pupils in a rectangular array and systematically giving each person a set of coordinates. By asking pupils to stand up if they satisfied inequalities like  $2x + 3y > 5$ , I involved all the pupils in thinking about the familiar coordinates in an unfamiliar way. The class teacher reported that “the activity maintained interest, was fun and entertaining and got across what I usually find pupils have difficulty with very clearly. The physical nature of the task helps keep interest and minds awake!”

The task should not be a passive routine, such as factorizing fifty similar-looking equations. Practice may be important to develop skill, but from this perspective the less routine there is in it, the more learning is likely to be achieved. In writing of the importance of a firm conceptual understanding, HMI state that “. . . progress in pupils’ mathematical understanding is more important than progress in the performance of skills. In fact, when the early stages of learning are firmly established subsequent progress can take place more quickly and confidently” (HMI 1985, 36).

## Reflection

ON COMPLETION OF AN ACTIVITY, A REVIEW OF THE learning achieved during the activity can be most helpful in assisting the learners to integrate their new or revised theories and expectations with their other mental systems.

“Activity per se is not a guarantee of mathematical learning” (Goodchild 1992, 24). Goodchild goes on to consider



Surprise and  
cognitive  
conflict can  
be useful to  
generate  
interest

the importance of reflection and “interpretation,” which he sees as a mechanism for making sense of the learning activity and for locating it in a wider framework of meaning and purpose. Alternatively, it may be seen as a process in which new personal theories are created or existing personal theories modified and in which new expectations may be created. The teacher’s role in this process is to ensure that there is time for such reflection, and to provide a mechanism to ensure that reflection occurs. The classroom organization of reflection may involve pupils in writing, or it may involve structured discussion with other pupils or with the teacher.

“When asked for the connections between practical work and the symbolic statement of rule, the children’s best reply was that one was a quicker route to the answer than the other. Nobody mentioned that the practical experience provided the data on which the formula was built. The teachers did not stress why this procedure was being followed, nor emphasize the generalizability of the rule and thus the advantage of accepting it” (Hart 1989, 139).

Within the theoretical framework outlined above, it may be seen as appropriate to ask pupils to be explicit about some new expectations. This obliges pupils to create both theories and expectations. To continue the calculator example: once pupils have amassed a number of calculations that give different answers on two calculators, they can be asked to predict the two answers for other calculations, to predict when the calculators will give the same answers and to predict when they will give different answers.

## Socialization

“KNOWLEDGE, FROM THE CONSTRUCTIVIST POINT OF view, is always contextual and never separated from the subject . . . to know also implies understanding in such a way that the knowledge can be shared with others and a community thus formed. A fundamental role is played by the negotiation of meaning in this interaction, which is of a social nature” (Moreno-Armella and Waldegg 1993, 657).

I do not believe that many pupils achieve this socialization of knowledge whilst working from individualized schemes. Even the pronunciation of key mathematical words (e.g., “sin” instead of sine) is missing, let alone the opportunity to discuss mathematical problem solving in depth. Socialization cannot be delegated to textbooks.

Having developed an understanding of a mathematical concept, there is a need for pupils to be able to communicate effectively about it. For this [requirement], there is often a need to know, understand and use the appropriate language

and terminology and to adopt the standard conventions. In other words, there is a need to socialize the personal and private understanding. The teacher has a role here in ensuring that the need for communication is apparent, that activities incorporate discussion work to facilitate such communication and that pupils are helped to become aware of conventional language and notations. “The use of discussion as a technique for teaching centres on the fact that it is above all else a means of escaping from our own individual perceptions of the world, with all their circumstances and boundaries into which we would otherwise be locked. It adds to the richness of understanding and enables us to make contact with the minds of others in the most direct way possible” (Van Ments 1990, 17). During the socialization process, the individual is being expected to negotiate a shared meaning with the teacher, peers, external examiners and textbooks largely by coming to grips with conventions and conventional language. If all pupils in a group are encouraged to communicate about their mathematical activity, this [approach] can provide a richer learning environment for each individual, as well as begin the process of negotiating shared meanings and socializing the new knowledge.

## Conclusion

IN THIS ARTICLE, I HAVE CONSIDERED A CONSTRUCTIVIST framework for the analysis of mathematical learning activities. This analysis suggests that mathematical activities are not enough to achieve learning by themselves; they need to be carried out with a consideration of aspects of presentation, the nature of the pupils’ mental activity, the need to ensure pupil reflection and the achievement of socialization of the learning.

## References

- Ahmed, A. *Better Mathematics*. London: Her Majesty’s Stationery Office (HMSO), 1987.
- Goodchild, S. “Active Learning, Reflection and Interpretation.” *Mathematics Education Review* 1 (1992): 24–29.
- Hart, K. “There Is Little Connection.” In *Mathematics Teaching: The State of the Art*, edited by Paul Ernest. London: Falmer Press, 1989.
- HMI. *Mathematics from 5 to 16*. London: Her Majesty’s Stationery Office (HMSO), 1985.
- Kyriacou, C. “Active Learning in Secondary School Mathematics.” *British Education Research Journal* 8 (1992): 309–18.
- Moreno-Armella, L., and G. Waldegg. “Constructivism and Mathematical Education.” *International Journal of Mathematics Education, Science and Technology* 24 (1993): 653–61.
- NCC. *Non-Statutory Guidance*. London: Her Majesty’s Stationery Office (HMSO), 1989.
- Smith, Jim. “Active Learning of Mathematics.” *Mathematics Teaching* (March 1996): 30–31, 35.
- Van Ments, M. *Act of Discussion in Learning*. London: Kogan Page, 1990. ▲