

Improving the Mathematical Content Knowledge of General and Special Educators: Evaluating a Professional Development Module That Focuses on Number Sense

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Valerie N. Faulkner¹ and Chris R. Cain²

Abstract

Student performance in mathematics has been linked to the mathematical knowledge of the teacher. Based on this finding, a 5-day professional development module was created to improve teachers' mathematical knowledge and their understanding of number sense. We found no difference prior to the professional development in mathematical content knowledge for teaching mathematics (CKTM) between special education teachers (at the K-12 level) and general education teachers (K-6). Results revealed that participating teachers made significant gains in mathematical CKTM. Implications and recommendations for professional development in mathematics are provided.

Keywords

professional development, mathematics, number sense, mathematical knowledge for teaching

The need for better and more effective professional development and teacher preparation in mathematics has been demonstrated by many studies indicating poor teacher understanding of mathematical topics in the United States (see for example, Ball, Bass, & Hill, 2004; Knuth, 2002; Ma, 1999). In the past, a typical solution for poor teacher understanding would be to require teachers to study more mathematics, including additional coursework (Ball, 2005). Additional coursework, however, often fails to provide a focus on specific content knowledge for teaching mathematics (CKTM), which has been found to be an effective element in professional development for teachers (Garet, Porter, Desimone, Birman, & Suk Yoon, 2001; Hill & Ball, 2004; Hill, Rowan, & Ball, 2005). Increased instructor knowledge of the mathematical content they

teach has an impact on student performance (Hill et al., 2005). Because this appears to be a critical factor in improving student performance, a logical next step is to begin to implement and evaluate professional development designed to improve teacher knowledge specific to the area of mathematics (Hiebert & Stigler, 2000; Stigler & Hiebert, 2004). Recently, Hill and Ball (2004) evaluated the effectiveness of the California Mathematics Professional Development Institute (CMPDI), utilizing the CKTM measures. CMPDIs are

¹North Carolina State University, Raleigh, USA

²Mars Hill College, NC, USA

Corresponding Author:

Valerie N. Faulkner, Department of Elementary Education, North Carolina State University, 317-K Poe Hall, Campus Box 7801, Raleigh, NC 27695-7801, USA.
Email: Valerie_faulkner@ncsu.edu

1- to 3-weeklong summer training programs that represent a large-scale effort to improve teacher knowledge as a way to impact student performance in mathematics. A focus on mathematical analysis, reasoning and communication, and the length of training were found to predict teacher learning (Hill & Ball, 2004). It is becoming increasingly clear that effective professional development to deepen teachers' understanding of mathematics should be part of the equation to improve mathematical learning in students.

Unfortunately, most professional development curricula have been found to be ineffective (Guskey, 2002). Therefore, it is important to find ways to analyze the effectiveness of professional development modules and allocate resources accordingly. There is qualitative evidence that strong teacher understanding of mathematics has an impact on appropriate classroom dialogue (Ma, 1999) as well as quantitative evidence that it has an impact on student gains in standardized measures (Hill et al., 2005). There is also evidence that teachers' attitudes about their instructional practices will change only after they see an impact on student performance (Guskey, 2002). One must provide professional development that (a) increases teacher understanding of mathematics within the content area and (b) increases the likelihood that teachers will habitually communicate mathematics more coherently and effectively. Hill and Ball (2004) found that a focus on mathematical analysis, proof, and communication led to higher gains in teacher learning. Specifically, the authors cite the opportunity to "engage in mathematical analysis, reasoning, and communication" as a critical component of this focus (p. 343). This was more important than the mathematical topics covered, the length of the training, group size, or the teachers' stated desire to learn. While this finding is promising, Hill and Ball conclude that there is a persistent "need to probe more carefully into the content of professional development and to identify curricular variables associated with teachers' learning" (p. 343). In the current study, we evaluate the North Carolina Foundations of Mathematics

Training (NCFMT) professional development model in an effort to identify elements of professional development that appear to be salient to improving CKTM. Throughout this article, we will use the term *content knowledge* to mean here specifically, the mathematical content knowledge teachers need to teach mathematics.

Results of The International Math and Science Studies (TIMSS) have stimulated discussion regarding performance of students in the United States (Gonzalez et al., 2004; Hiebert & Stigler, 2000; Kimmelman et al., 1999; National Center for Education Statistics [NCES], 1999; Silver, 1998) and continue to put pressure on our need to make effective changes. The mathematical knowledge of teachers needs further investigation so that we better understand how student success is tied to teacher implementation choices regarding problem sets, questioning techniques, and math connections (Hill et al., 2005; Stigler & Hiebert, 2004). These connections may be related to the coherence, and thus the effectiveness, of a given lesson. As a whole, classrooms in the United States stand in contrast to countries where "students are given the opportunity to infer coherence across the episodes that constitute their experience in mathematics class" (Stigler & Perry, 1990, p. 349; and discussed in Confrey, 2007).

The idea of a coherent mathematical message is another way of characterizing efforts to develop students' "number sense." Oftentimes, the issues surrounding mathematical content in the general education mathematics community emphasize the need to develop student mathematical thinking that demonstrates number sense that is fluent, flexible, and guided by meaning (Fennell & Landis, 1994; Mathematics Learning Study Committee, National Research Council, 2001). Other discussions provide specific examples of activities that develop number sense (Collison, Schwarz, & Collison, 2006). Ironically, this mathematical coherence or number sense has not been clearly defined for teachers' instructional purposes, despite the growing literature on this topic. This difficulty with coherence could be a direct result

of the way many of us were taught mathematics as students because many teachers are products of the same system we seek to improve (Ball, 2005). We suggest that coherence to develop number sense includes connecting mathematical ideas that teachers in the United States often treat as separate topics. For instance, connecting a discussion of exponents to the three dimensions of the world we live in (“squaring” and “cubing”) as well as to the idea of units and unit sizes (“square unit,” “cube unit”), which then connects to measurement.

In the current study, we evaluate general as well as special educators to understand this issue from a broader school perspective. Efforts to improve lower performing students’ mathematical abilities have traditionally focused on pedagogical issues such as providing metacognitive support (Jitendra, Hoff, & Beck, 1999) or a concrete to representational to abstract (CRA) presentation (Mercer & Miller, 1992; Miller & Mercer, 1994). More recent efforts to support the struggling student consider issues of reform-based instruction, utilizing “big ideas” to organize instruction and estimation (Baxter, Woodward, & Olson, 2001; Gersten & Chard, 1999; Woodward & Montague, 2002), but these have not included clear directions regarding recommended professional development or the preservice training of special education teachers.

Teachers’ Mathematical Knowledge and Special Education

There has been an increased focus on mathematics in the special education community due to No Child Left Behind and the Individuals With Disabilities Improved Education Act. This intensified attention has focused primarily on the characteristics and needs of the struggling learner, including issues regarding assessment and intervention (Cawley, Parmar, Foley, Salmon, & Roy, 2001; Gersten, Clarke, & Jordan, 2007; Kroesbergen & Luit, 2003), implications of a standards-based instructional emphasis for student learning (Baxter et al., 2001; Cawley

et al., 2001; Maccini & Gagnon, 2002), and general recommendations for the struggling student by the National Math Panel (Gersten et al., 2008).

In spite of the current efforts within the mathematics education community to respond to the findings in the TIMSS studies and the mounting evidence regarding the importance of mathematical knowledge in teachers, little or no attention has been paid to developing the mathematical knowledge of special educators and their general education colleagues who work with low-performing students. In fact, certified K-12 special education teachers often have taken the same mathematics methods courses as K-6 general education teachers. This is problematic for two reasons: (a) These teachers are expected to help the struggling math student using the same sets of tools as the general education teacher and (b) they are often dealing with a higher level of content in middle and high school than what their teacher training has prepared them to teach.

Only recently has number sense been specifically defined for teachers within the special education community in the form of an instructional model (Faulkner, 2009). Teachers need specific support in “unpacking” mathematics to ensure that they are spending time with students on discussions and activities that will improve number sense (Ball et al., 2004). This may be particularly true for special educators who are less familiar with the current requirements of mathematics curricula as compared with other subject-specific instructors. In one study, more than half of special educators indicated that they were not aware of the National Council of Teachers of Mathematic Standards (Maccini & Gagnon, 2002). As a result of the study, Maccini and Gagnon (2002) have called for intensive teacher training in mathematics for special educators.

Present Study

The primary purpose of the current study was to measure the effects of a 40-hr professional development course designed to improve educators’ mathematical knowledge. This

training, called the NCFMT (Faulkner, Cain, Hale, & Doggett, 2006), focuses particularly on a model for number sense designed to develop teachers' mathematical content knowledge and ability to deliver a coherent mathematical message through instruction (Cain, Doggett, Faulkner, & Hale, 2007; Faulkner, 2009). Locally, state administrators are required to make short-term policy decisions on how to utilize federal funds to support teachers by increasing research-based practices. Evaluating whether the NCFMT, with its emphasis on the Components of Number Sense, had an impact on participants is a responsible method for making research-based decisions regarding the use of funds. Furthermore, it is particularly appropriate to utilize an evidence-based model to evaluate the success of professional development given the recent emphasis and increased onus on teachers to use evidence-based classroom practices.

A secondary purpose for this research was to understand the mathematical content knowledge of educators who support students with special needs. A recent study found that not only are less than half of special educators familiar with current mathematics standards, those special educators who are familiar with the National Council of Teachers of Mathematics (NCTM) standards are less confident in teaching them than their general education peers (Maccini & Gagnon, 2002). It would be useful to understand whether special educators are, indeed, weaker in mathematical content knowledge than their general education peers. This question has policy implications in that if special educators are shown to have much weaker content knowledge, funds should be used toward improving this. It is unlikely that teachers with weak mathematical content knowledge will be able to effectively use texts and materials.

Therefore, the purpose of this study was to examine the following research questions:

Research Question 1: Does the NCFMT have a significant impact on teacher knowledge for teaching mathematics?

Research Question 2: Is there evidence of a difference in content knowledge between special educators and general educators?

We hypothesized that the NCFMT, designed specifically to support teachers' thinking about mathematical connections and number sense, would have a significant impact on teachers' mathematical content knowledge. Given the current lack of information regarding which specific instructional elements and activities influence this knowledge, our prediction was based on the growing emphasis on the general importance of number sense found in recent literature. The competing hypothesis, however, that this training would not significantly affect teacher knowledge, also had merit. In the California study cited earlier, the CMPDI did not emphasize number sense per se, and the authors did not note it as a predictor for teacher growth. They found that the duration of the training (up to 3 times as long as the NCFMT) was a predictor for growth, which also argues against our hypothesis.

The purpose of this study was to determine whether the NCFMT was effective and not whether any particular trainer was effective. The NCFMT, as designed, is a train-the-trainer model. Of practical concern with this model is whether different trainers are able to maintain fidelity in implementation so that positive results can be reliably predicted and replicated. It was built into our hypothesis that the training would yield similar results regardless of who implemented the training. We consider this aspect of the study critical because of the concern that a specific trainer, and not the NCFMT itself, may have been responsible for any findings. This component of the study helped us to disentangle this confounding variable and is treated below in the "Method" section.

With regard to the initial mathematical knowledge of special educators, we speculated, based in part on the literature outlined earlier, that special educators would have less mathematical content knowledge than their general education peers. This prediction was consistent with Maccini and Gagnon's call for

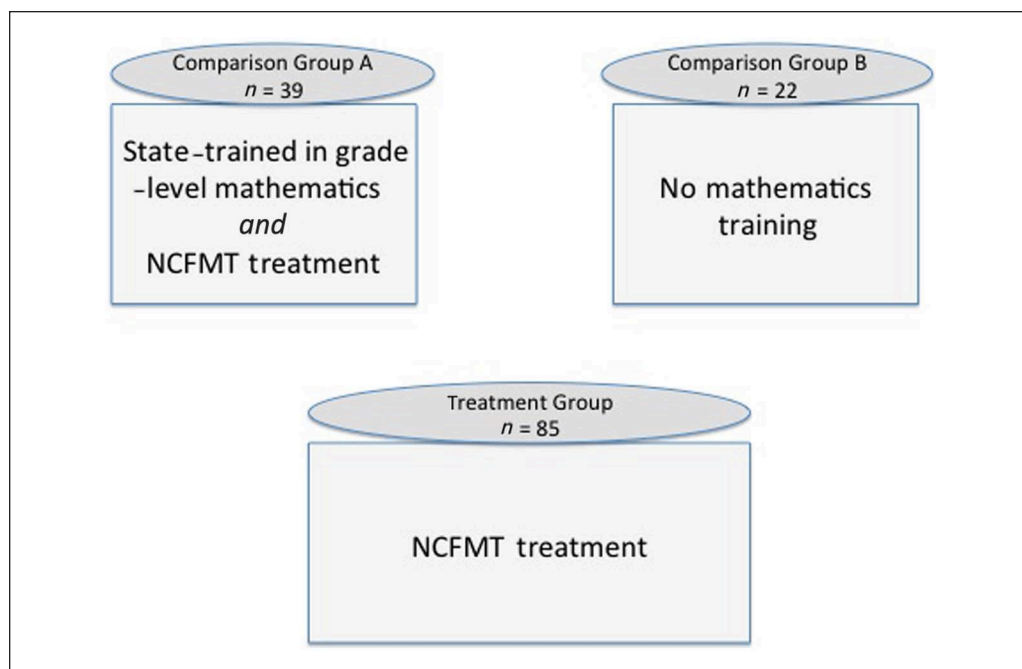


Figure 1. Treatment group and comparison groups.
Note: NCFMT = North Carolina foundations of mathematics training.

intensive teacher training for special educators based on their findings regarding special educators' general lack of knowledge and confidence regarding NCTM recommendations (Maccini & Gagnon, 2002).

Method

Participants

The groups involved in the study included the Treatment Group ($n = 85$), Comparison Group A ($n = 39$), and Comparison Group B ($n = 22$). All three of these groups were trained by Trainer A.

The Treatment Group of participants ($n = 85$) included K-12 mathematics general education and core subject teachers ($n = 69$) as well as special education teachers certified in K-12 instruction ($n = 16$). The Treatment Group participated in a districtwide NCFMT in an effort to become highly qualified and/or knowledgeable about appropriate scientific research-based instruction and pedagogically skilled in effectively instructing students in mathematics.

The teachers in this district had not had any districtwide professional development in mathematics for the past 7 years. The NCFMT was done in several sets, with no group exceeding 25 participants at any given time.

Comparison Group A ($n = 39$) comprised K-12 teachers who participated in the NCFMT as well as a separate state-led mathematics professional development over the past 2 years. This separate state-led training was offered within their district and focused on research-based instruction at the specific grade levels. These teachers were given the NCFMT after exposure to this 2-year professional development program. Comparison Group A was divided into two groups with numbers of 20 and 19 for training purposes.

Comparison Group B ($n = 22$) comprised K-12 teachers who participated in reading training conducted by a nearby school system. They received research-based instruction in reading during their 5-day professional development. This group of teachers did not receive any specific mathematics training throughout the duration of this study (Figure 1).

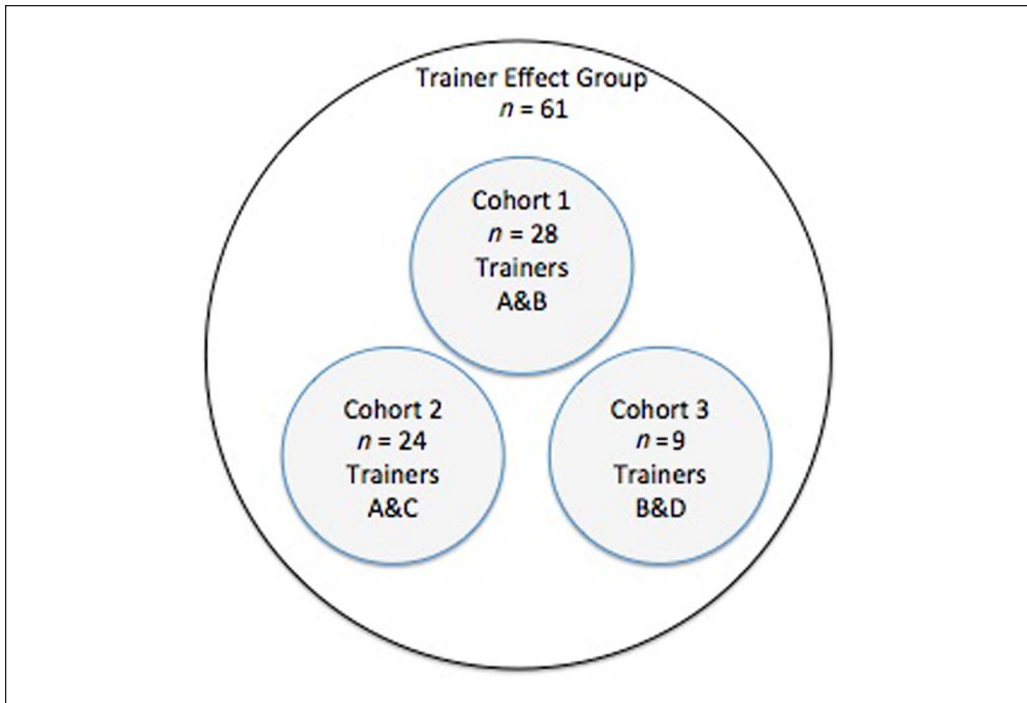


Figure 2. Composition of Trainer Effect Group by trainers.

A major consideration in a study such as this is whether one is actually measuring the effectiveness of a particular *trainer* or the effectiveness of the *training curriculum*. To increase confidence that we were actually measuring the effectiveness of the NCFMT curriculum itself, we ran an analysis comparing results of the Treatment Group (trained with NCFMT by Trainer A) with a Trainer Effect Group who is not in the primary analysis described earlier but used to disentangle the trainer effect confound. These participants were also trained in the NCFMT but by a variety of trainers (Trainer A, as well as Trainers B, C, and D). The four trainers responsible for content in the Trainer Effect Group were Trainer A, the second author herein and coauthor of NCFMT; Trainer B, coauthor of the NCFMT; Trainer C, the first author herein and coauthor of NCFMT; and Trainer D, a “second generation” trainer who has become certified since the inception of the model. These authors shared content training equally in their respective cohorts within the larger Trainer Effect Group (Figure 2).

The participants in the Trainer Effect Group consisted of an additional 61 K-12 special education teacher participants (Figure 2).

Special educators ($n = 77$) and general educators ($n = 69$) were drawn from the Treatment Group and the Trainer Effect Group to investigate the possible differences in content knowledge between these two groups of educators (Figure 3).

Measure

Mathematical knowledge for teaching was measured using CKTM—Number and Operations forms A and C. These forms were developed at the University of Michigan specifically for the purpose of capturing and measuring the mathematical skills teachers need to be effective with their students. Forms A and C have been statistically equated for use as pretest and posttest measures (Hill, 2008). They are multiple-choice, with one to four response items per question stem. Form A (pre-test) has 13 question stems and 26 total items with a reported reliability measure of

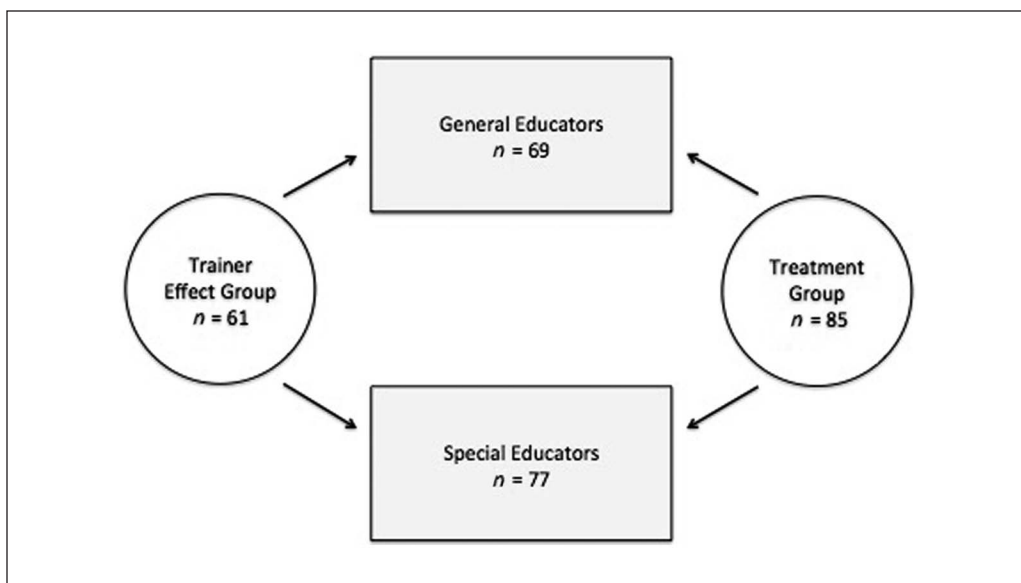


Figure 3. General and special educators with pretest data from Treatment and Trainer Effect Groups.

.72 and form C (post-test) has 14 question stems and 23 total items with a reliability measure of .71 (Hill & Ball, 2004).

All raw scores were converted to item response theory (IRT) scale scores. IRT scale scores are given in standard deviation units. As such, a CKTM score of 0, on a scale of -3 to 3, represents average ability. The benefit of using IRT in testing is that scores from different batteries of tests can be compared. In equating CKTM test scores, evaluators can examine growth (i.e., changes in ability) due to interventions (Hill & Lubienski, 2007). For a more extensive treatment of the development and statistical analysis of these measures, see Hill and Ball (2004); Hill, Ball, and Schilling (2008); and Hill et al. (2005).

Both authors attended training for the use of the CKTM measure through the University of Michigan and both were curriculum writers and trainers for the NCFMT. None of the curriculum authors/trainers gained access to the CKTM measure until after the NCFMT curriculum was written and training was not geared toward the content of the measure. As recommended by the developers, the CKTM measure was used in a pretest and posttest capacity to evaluate the impact of the NCFMT.

The pre-test (Form A) was given during the morning of the first day of training and the post-test (Form C) was given late in the morning of the fifth day of training. Participants were given 50 min to complete the measure.

Content validity was addressed by the authors of the measures through comparisons of item topics with the Principles and Standards of School Mathematics developed by the NCTM (Hill & Ball, 2004). The pre- and post-tests serve the following three functions: (a) to reflect the knowledge teachers use in teaching—the content they actually teach as well as the special knowledge they must have to teach mathematics to their students; (b) to situate test items in the context teachers face in classrooms—examining textbook definitions for accuracy, designing classroom tasks, and evaluating student statements; and (c) to discriminate between knowledge levels among teachers without representing particular views about how mathematics should be taught. The CKTM measures teachers' conceptual understandings of mathematics as well as their understandings of mathematical procedures, see Hill, H.C. (2008).

Because the measure included content knowledge of basic operations and elementary

number operations, it was well aligned with the goal of the NCFMT to increase fundamental mathematical content knowledge of teachers who work with students with mild disabilities and other struggling learners.

Intervention

The NCFMT staff development module was written by two secondary mathematics educators and two special educators. The purpose of the training was to provide meaningful professional development to teachers that would translate into stronger classroom practices for those who instruct students with special needs in the North Carolina Standard Course of Study for mathematics. The authors of this article served as the special educators on the NCFMT author team. The professional development is a 5-day training course designed to be provided over 2 to 3 months and can be implemented during the school year or over the summer. The workshop is lecture-based, with generous opportunities for participant discussion. Mathematical activities and explorations were also built into the format. While participants were responsible for readings on their own, most activities and assignments were group work.

Individuals in this study participated in the 5-day intervention over the course of 8-10 weeks during the spring of 2011.

Rationale for Intervention. The NCFMT focuses on developing an understanding of the fundamentals of mathematics as well as classroom habits that emphasize number sense for a coherent presentation of mathematics. The desired outcome of the training is to support teachers in utilizing more than just the “trappings” of conceptual instruction (the use of manipulatives, for instance) so that they might present math in a manner that increases the coherence of the mathematics presented and the quality of the interaction between the teacher and the students (see, Confrey, 2007; Stigler & Perry, 1990 for further discussions of classroom coherence). The authors of the training collaborated to develop a model of number sense to anchor the instructional work of the training. This model, the Components

of Number Sense (Faulkner, 2009; Faulkner, et al., 2006), was used to provide teachers with a greater understanding of the connections that need to be made for students through the habits of their instruction (Faulkner, 2009). It was created because the instructional underpinnings of number sense development have not been outlined for the teaching community. Even those who research the construct of number sense continue to refer to it as “difficult to define but easy to recognize” (Gersten, Jordan, & Flojo, 2005). Kalchman, Moss, and Case (2001) described number sense as follows:

The characteristics of good number sense include: (a) fluency in estimating and judging magnitude, (b) ability to recognize unreasonable results, (c) flexibility when mentally computing, (d) ability to move among different representations and to use the most appropriate representations. (p.2)

These attempts to define number sense are not easily translated into classroom practice. The Components of Number Sense as outlined in the NCFMT model are quantity and magnitude, numeration, equality, base 10, forms of a number, proportional reasoning, and algebraic and geometric thinking (Figure 4). These are connected by the use of accurate mathematical language emphasizing these components. During the workshop, the Components of Number Sense model was utilized to (a) increase teacher knowledge about mathematics and (b) direct teacher movement away from strictly procedural lessons and toward a habit of emphasizing discussions that develop number sense in their students. Participants were shown how these components function within the system of mathematics through lectures, readings, and mathematical explorations. Homework assignments connected workshop ideas to instructional practice and were also an integral part of the training, with several assignments specifically designed to give teachers practice in thinking of mathematics through this lens (see appendix: NCFMT Training Outline, including readings, samples of trainer notes, and sample homework assignments).

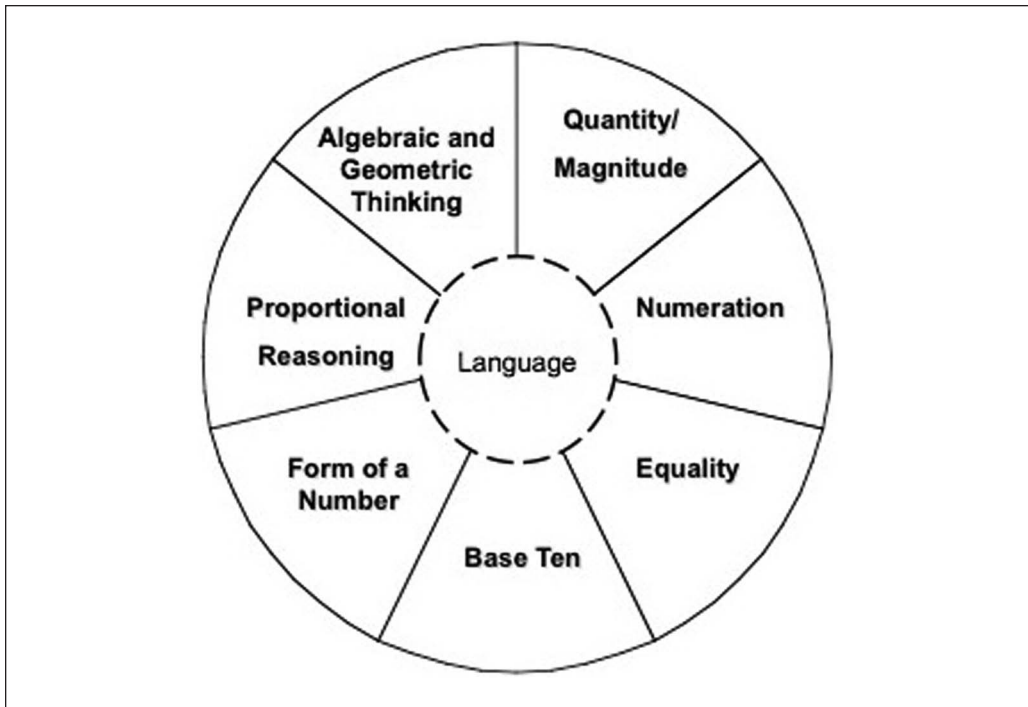


Figure 4. The Components of Number Sense.

The NCFMT approach is consistent with the Common Core Standards as it encourages teachers to communicate mathematics through meaningful conceptual activities. The importance and effectiveness of directly teaching these concepts is also emphasized as an important method for teaching the struggling learner.

Results

The analysis and results are organized by research question, with discussion to follow in a separate section.

Research Question 1: Does the NCFMT have a significant impact on teacher knowledge for teaching mathematics?

To evaluate the differences between preintervention and postintervention test scores, we examined the Pre–Post \times Group interaction by way of a 3×2 mixed ANOVA (Factor 1: Three groups including the Treatment Group [$n = 85$], Comparison Group A teachers who

received the intervention [$n = 39$], and Comparison Group B who did not receive the treatment [$n = 22$]; Factor 2: Repeated measures, including pre- and post-tests).

We used an ANOVA to examine posttest differences between Treatment teachers ($n = 85$), Comparison A teachers (received treatment, $n = 39$), and Comparison B teachers (did not receive treatment, $n = 22$). Measures of effect size include partial eta-squared for the ANOVA. For post hoc comparisons, we used the Bonferroni adjustment to calculate 95% confidence intervals for the mean differences between groups.

The repeated measure interaction charts (Figure 5) illustrate the significant interaction effect in the study. Rising slopes for the Treatment and Comparison A groups are indicative of growth. The Comparison B group does not show an increase in performance and, in fact, demonstrates a slight decline. The different trends in pretest and posttest scores for the groups indicate that the treatment and comparative groups were

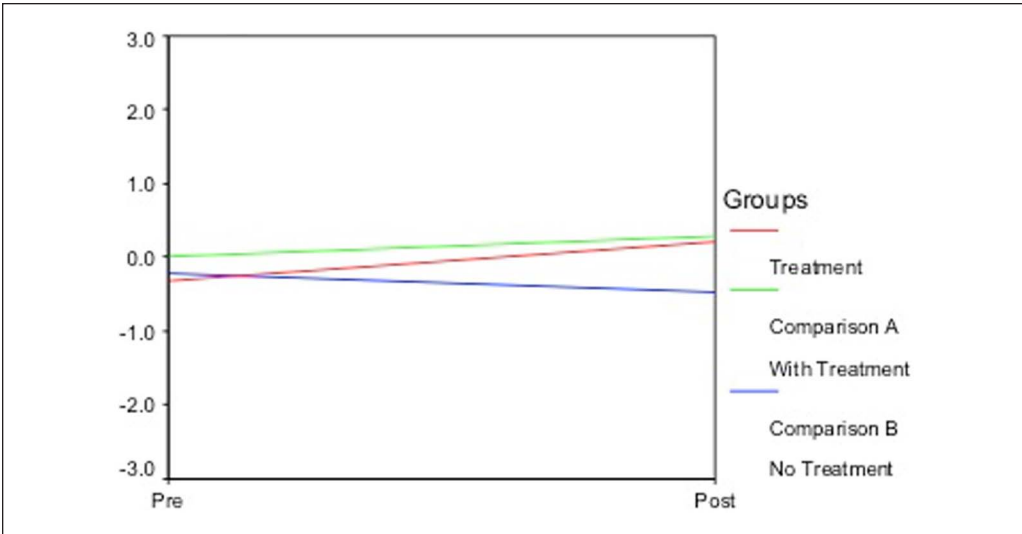


Figure 5. Repeated measure interaction effect on pre- and post-tests.

Note: $N = 146$ (Treatment = 85; Comparison A = 39; Comparison B = 22).

Table 1. Mathematical Knowledge for Teaching, Mean Comparisons.

| Groups | Pre-test | Post-test | Difference |
|-----------------------------------|----------|-----------|------------|
| Treatment | -0.315 | 0.223 | 0.538 |
| Comparison A (received treatment) | 0.02 | 0.298 | 0.278 |
| Comparison B (no treatment) | -0.215 | -0.457 | -0.242 |

Note: $N = 146$ (Treatment = 85; Comparison A [received treatment] = 39; and Comparison B [no treatment] = 22).

affected differently and support the claim that the intervention had a positive impact on post-test performance.

Table 1 reports the conditional means for pretest and posttest scores for the Treatment Group and two Comparison Groups. For example, the pretest mean for the Treatment Group was -0.315 and the posttest mean was 0.223 . The change in scores resulting from the NCFMT professional development was 0.538 , just over half of a standard deviation unit. This interaction was significant, $F(2, 143) = 9.388, p < .005, = \eta^2_{\text{partial}} 0.116$. The partial eta-squared effect size was small for the interaction.

The mean difference in pretest and posttest scores of the Treatment Group teachers was just over half of a standard deviation (0.538). The paired-samples t test of this difference was significant, with a moderately large effect

size, Cohen’s $d = 0.58$, indicating a level of success with the NCFMT intervention.

The posttest means for the Treatment Group (Figure 6) and the two Comparison Groups were also analyzed using posttest ANOVA and post hoc comparisons. The mean posttest score of Treatment teachers, for example, was 0.223 . While the partial eta-squared effect size was small, the ANOVA for comparing posttest scores between groups was highly significant, $F(2, 143) = 5.862, p = .004, \eta^2_{\text{partial}} = 0.076$.

The post hoc comparisons for Treatment Group versus Comparison Group A, Treatment Group versus Comparison Group B, and Comparison Group A versus Comparison Group B can be found in Table 2. This table reports the mean differences for these comparisons as well as the 95% confidence intervals. For example, the mean difference

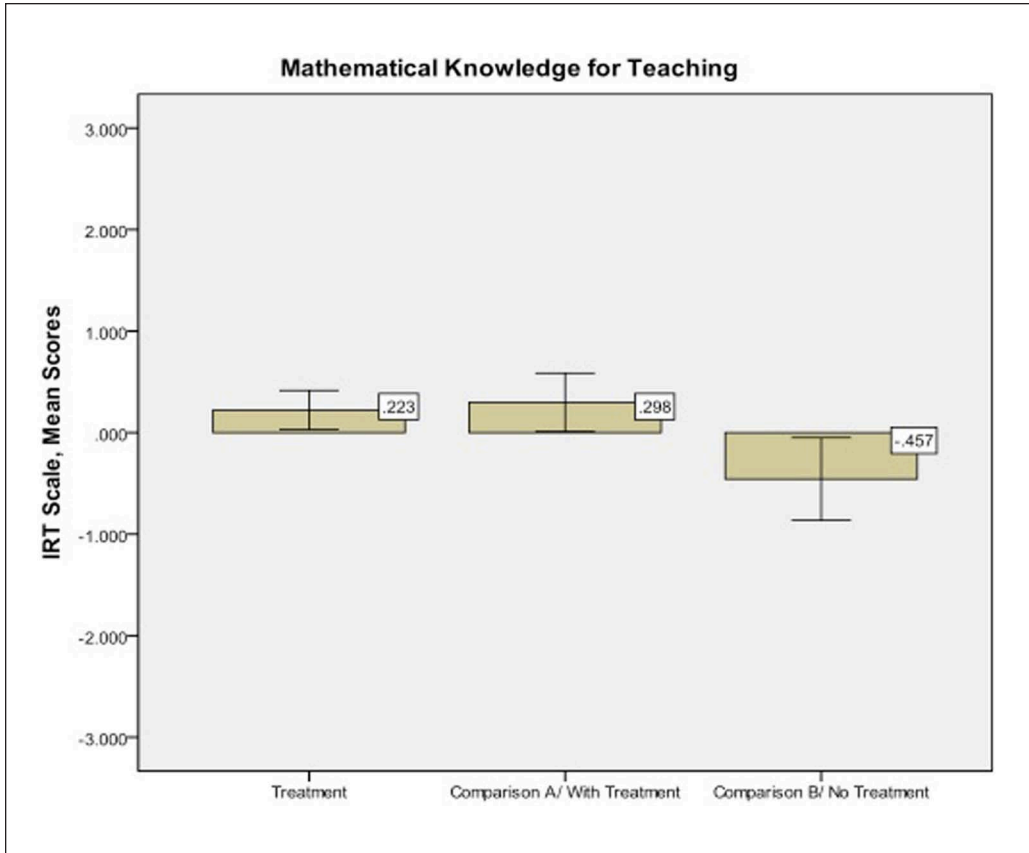


Figure 6. Posttest comparisons.

Note: $N = 146$ (Treatment = 85; Comparison A = 39; Comparison B = 22). IRT = item response theory. Error bars indicate 95% confidence intervals.

Table 2. Significant Post Hoc Differences (Bonferroni).

| Comparisons | Mean differences | 95% confidence intervals |
|---------------------------|------------------|-----------------------------|
| Treatment–Comparison A | –0.075 | Inconclusive |
| Treatment–Comparison B | 0.680 | [0.0161 < δ < 1.199] |
| Comparison A–Comparison B | 0.755 | [0.177 < δ < 1.333] |

Note: $N = 146$ (Treatment = 85; Comparison A = 39; Comparison B = 22).

between Treatment and Comparison B posttest scores was 0.680. The confidence interval for this comparison indicates that the mean difference between the two groups could be as little as 0.02 or as great as 1.2 points on the IRT scale.

To understand whether the effects demonstrated above were truly due to the NCFMT training rather than a particular trainer, an

analysis was run comparing results of NCFMT training done by other trainers in other parts of the state. This group, the Trainer Effect Group, is described in the “Participants” section above (see Figure 2).

In testing for a trainer effect, we examined the Pre–Post \times Group interaction by way of a 4×2 mixed ANOVA. There were four groups in this repeated measures analysis, the

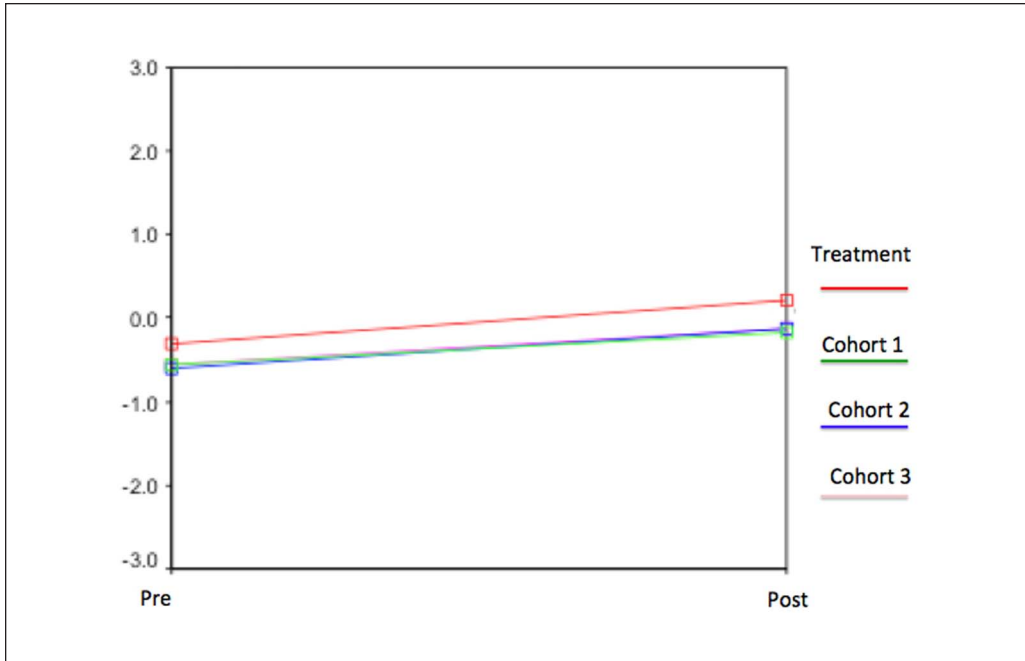


Figure 7. Treatment versus Trainer Effect Groups.

Treatment Group ($n = 85$) and teachers from three other groups who received the intervention (Cohort 1, $n = 28$; Cohort 2, $n = 24$; and Cohort 3, $n = 9$).

Figure 7 illustrates the noninteraction for the Pre–Post \times Group ANOVA. Rising slopes for all groups show growth, with the Treatment Group showing the highest pretest and posttest scores. The pretest mean for the Treatment group was -0.315 and the posttest mean was 0.223 (Table 1). The growth over time was 0.538 , just over half of a standard deviation. While the pretest and posttest means are lower for the Trainer Effect cohorts, the demonstrated growth was consistent across all groups and thus, there was no interaction effect, $F(3, 146) = 0.272, p = .845$. This indicates that there is no trainer effect and provides evidence for the claim that the growth is related to the training itself rather than particular trainers.

Research Question 2: Is there evidence of a difference in content knowledge between special educators and general educators?

In analyzing preintervention content knowledge, we compared pretest scores of special and general education teachers using an independent samples t test. In this analysis, there were 77 special education teachers and 69 general education teachers (see Figure 3). The mean preintervention score of special education teachers was -0.556 with a standard deviation of 0.787 . The mean preintervention score of regular education teachers was -0.275 with a standard deviation of 0.946 . The mean difference in scores was -0.281 with a standard error difference of 0.144 . The independent samples t test (equal variances not assumed) was inconclusive, $t(133) = -1.94, p = .055$. Given the inconclusive t test, there is no evidence of a difference between special and regular education teachers with regard to preintervention content knowledge.

Discussion

Participation in the NCFMT helped teachers make significant gains in knowledge as measured by the CKTM. Teachers participating

in the NCFMT exhibited gains at least as strong as those who participated in the CMPDI in 2001, where these measures were first used to evaluate the effectiveness of professional development. In that study, reported in 2004, teachers participated in 5, 10, or 15 days of training and demonstrated average gains of .47 logits. Teachers in this study demonstrated gains of .54 logits after 5 days of training held over the course of 8 to 10 weeks. Treatment Effect cohorts likewise saw considerable growth of between .40 and .47 logits. This finding is consistent with the initial hypothesis that a professional development emphasis on number sense would have an impact on teacher content knowledge.

Given the inconclusive *t* test, there is no evidence of a difference between special and general education teachers with regard to pre-intervention content knowledge. At the same time, the pretest scores of special educators were half of a standard deviation below the norm (-0.556). It may be of interest to compare special and general education teachers' posttest scores to see whether one group performed better than the other after the intervention. It is important to recognize that the special education group was primarily compared with elementary general education teachers, even though the special education teachers are certified to work with K-12 students with disabilities.

Implications

The NCFMT content was designed specifically to develop instructors' knowledge of the mathematics they teach by seeing mathematics through the lens of a well-delineated number sense. Effective gains in short periods of time are an important consideration given the cost of teacher training. The strong gains may be the result of the NCFMT scheduling format that spreads the training dates out over a 2-month period, the content of the NCFMT, or both. The gains made by teachers in this training may also indicate that the Components of Number Sense model is a useful tool in

teacher training. This utility may lie in the fact that it provides teachers with a cognitive model for number sense that serves to strengthen their understanding of mathematical connections and supports current efforts such as those espoused by the Common Core Standards. If this is in fact the case, this model may affect classroom instruction by increasing the mathematical coherence presented to students as described by Stigler and Perry (1990).

It is also valuable to have a tool designed to evaluate the effectiveness of professional development, and in this study, we extend the initial findings regarding the utility of the CKTM measure as reported by Hill and Ball (2004). Evaluation conducted by the North Carolina Department of Education verifies that special education students taught by teachers in the current study made substantially greater gains on End of Grade tests than peers taught by teachers who had not been trained. Although this evaluation was not a formal research study, the results are encouraging. Further research in this area will help to establish the connection between effective staff development, teacher gains in mathematical knowledge, and student achievement.

The question of whether or not second- and third-generation trainers maintain the fidelity and effectiveness of the initial training is essential due to the entrenched nature of the train-the-trainer model in the education community. Follow-up studies tracking the effectiveness of the NCFMT as delivered by district-level trainees are planned. These will provide valuable information regarding whether the gains reported herein can be attributed reliably to the NCFMT training as written and prescribed rather than to the idiosyncratic effectiveness of the trainers delivering them. It will also shed light on the capacity for the NCFMT to be used more widely as a relatively inexpensive and effective method of increasing teacher knowledge.

Recent law mandates that teachers are highly qualified. The spirit of this mandate is particularly demanding for special educators since they may teach several subject areas.

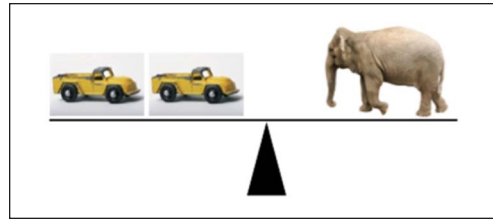
While the difference in knowledge between general and special educators was not found to be significant in this study, we provide possible evidence to support Maccini and Gagnon's (2002) call for "intensive teacher training" for special educators in the area of mathematics. The evidence here suggests that special educators do, indeed, have content knowledge that is below norm ($SD = -0.56$). Maccini and Gagnon also raise the issue of whether secondary students identified as having special needs are enrolled in challenging mathematics courses and/or are provided adequate support by special educators to succeed in these courses. At the same time, the results herein are particularly encouraging as special educators made substantial gains in all quartiles of performance once provided with the 5-day training studied herein.

Appendix

North Carolina Foundations of Mathematics Training

Course Topics and Readings

- Unit 1 Overview of NCFMT
Readings: (Ball & Cohen, 1996)
- Unit 2 Research on mathematics and the struggling learner
Readings: (Gersten, Jordan, & Flojo, 2005; Griffin, 2003)
- Unit 3 Profound understanding of fundamental mathematics
Readings: (Ball, 1992, 1993; Ma, 1999)
- Unit 4 The Components of Number Sense—overview
- Unit 5 Quantity/magnitude and numeration
Readings: (Mayer, 2003)
- Unit 6 Equality, base ten and form of a number
Readings: (Sizer, n.d.)
- Unit 7 Proportional reasoning and algebraic/geometric thinking
- Unit 8 Assessment
- Unit 9 Reflections
Readings: (Woodward, 2004)



Example of trainer notes for discussion during training: Equality and the importance of precise language.

Implementation choices: Language and accurate and consistent communication using a common language is a huge part of implementation choices. Let's look at the same situation in a different venue.

Are these equal? In what ways are they equal? Are there ways in which they are "not the same"?

These two are equal in weight, but they are not the same thing when we consider their physical dimensions.

We often think we are being more precise for kids by simplifying things when in fact we are communicating misconceptions. In a situation like this, the teacher's implementation choices should include statements indicating that they are the same in weight, but they are not the same thing.

Classroom Example. Second Grade—We are making an exchange of 10 with rods and blocks—Are they the same thing? Child already has a sense of sameness and replies "No." You are right—they are not the same, but they are equal in value—namely, they both equal 10 units.

This is what they found in The International Math and Science Studies (TIMSS) studies. The countries that are producing stronger math students are the ones where the teachers are making these kinds of habitual implementation choices. They are very aware of how language accurately communicates mathematical concepts. Are we laying down accurate mathematical language that will facilitate future learning and understanding of math as opposed to focusing on getting the right answer in this class, at this minute? We are

laying down synapses in the students' brains that will last a lifetime.

Example Assignments

Learning task for Unit 4. In a paragraph or two, discuss a mini-lesson around one of the Components of Number Sense. Write at least ten other questions that you could ask students to connect your lesson to the other Components of Number Sense. Remember that you are trying to “hook” them into what they already know about a particular concept to build an understanding of the new concept.

Discussion questions for Unit 6. In three paragraphs, describe how your understanding regarding base ten, equality/inequality and form of a number changed after exposure to this unit?

Discussion questions for Unit 8

1. Analyze the situation below and come up with a hypothesis. Also, describe some of the strategies you would suggest the teacher use to remedy the problem.

Susan, a third-grade student identified as having learning disabilities, is being provided additional help with her arithmetic skills by a resource teacher. The teacher has taught Susan how to compute three digit multiplication problems (324×435) using the type of sound mathematical instruction discussed in this training. The teacher taught Susan in a small group situation, using paper/pencil tasks. However, she found that Susan had not mastered the multiplication skills. Based on your expertise, what questions should you ask yourself as you develop an assessment plan?

2. Come up with three possible causes for Susan's problem. How does the response to intervention process fit into Susan's assessment and intervention? What data would you collect, and how would you use this data?

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Author Biographies

Valerie N. Faulkner is a teaching assistant professor at North Carolina State University in the Elementary Education department where she also earned a master's degree in special education and a doctorate in curriculum and instruction with a focus in mathematics education.

Chris Cain is an associate professor at Mars Hill College where he is the Coordinator of MHC's Master of Education, Academically & Intellectually Gifted, and Integrated Education Programs.