MAT 195 – Spring Quarter 2002
TEST 2 - Answers

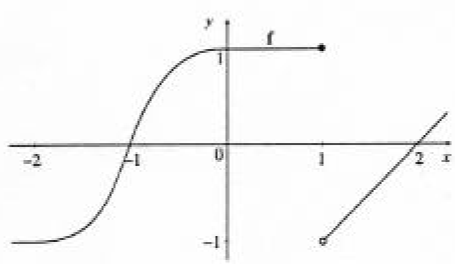
NAME____
Show work and write clearly.

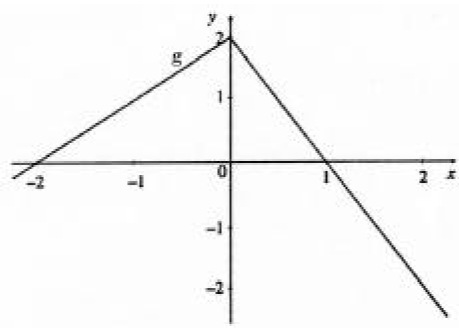
1. The displacement (in meters) of an object moving in a straight line is given by $s = 1 - \frac{t}{4} + 2t^2$, where t is measured in seconds.

a. Find the average velocity over the following time periods:
(i) [1, 2]
ANS: [1, 2] means the interval from t = 1 to t = 2. Then $s(1) = 2.75$ and $s(2) = 8.5$. The average velocity is the slope of the secant line between the points (1, 2.75) and (2, 8.5) which is
$\frac{8.5 - 2.75}{2 - 1} = 5.75$ m/s.
(ii) [1, 1.5]
ANS: Refer to the answer above. $s(1.5) = 5.125$ and the average velocity is 4.75 m/s.
(iii) [1, 1.1]
ANS: Refer to the answer above. $s(1.1) = 3.145$ and the average velocity is 3.95 m/s.

b. Estimate the instantaneous velocity (to 4 decimal places) when t = 1. Explain.
ANS: There are several ways to answer this question. One way is to find the slope of the secant line when the two points are very close: (1, 2.75) and (1.0001, 2.75038) which is 3.8 m/s.

2. Referring to the graphs below, find each limit, if it exists. If the limit does not exist, explain why.





a. $\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{1}{2}$

b. $\lim_{x \to 1} [f(x) \cdot g(x)]$ DNE because $\lim_{x \to 1} f(x)$ DNE

c. $\lim_{x \to -1} \frac{g(x)}{f(x)}$ DNE because $\lim_{x \to -1} f(x) = 0$

d. $\lim_{x \to 2} [x \cdot g(x)] = 2 \cdot -2 = -4$

e. $\lim_{x \to -1} [f(x) + g(x)] = 0 + 1 = 1$

f. $\lim_{x \to 1^-} [x + f(x)] = 1 + 1 = 2$

g. $\lim_{x \to 1^+} \frac{g(x)}{f(x)} = \frac{0}{-1} = 0$

3. $f(x) = \begin{cases} \sqrt{3-x} & x \le 1 \\ x^2 & 1 < x < 3 \\ 27/x & x \ge 3 \end{cases}$

a. Evaluate each limit, if it exists. If the limit does not exist, explain why.

i. $\lim_{x\to 1^-} f(x) = \sqrt{2}$

ii. $\lim_{x\to 1^+} f(x) = 1$

iii. $\lim_{x\to 1} f(x)$ DNE because directional limits are not the same

iv. $\lim_{x\to 3^-} f(x) = 9$

v. $\lim_{x\to 3^+} f(x) = 9$

vi. $\lim_{x\to 3} f(x) = 9$

vii. $\lim_{x\to 9} f(x) = 3$

viii. $\lim_{x\to -6} f(x) = 3$

b. What is the domain of $f(x)$.
ANS: $(-\infty, \infty)$

c. Where is $f(x)$ discontinuous? Explain.
ANS: At $x = 1$ because the limit DNE.

d. Where is $f(x)$ *not* differentiable? Explain.
ANS: At $x = 1$ because the function is not continuous and at $x = 3$ because cusp.

4. Find the limits, algebraically.

a. $\lim_{x\to\infty} \dfrac{\sqrt{x^2 - 9}}{2x - 6} = \lim_{x\to\infty} \dfrac{\sqrt{1 - \frac{9}{x^2}}}{2 - \frac{6}{x}} = \dfrac{\sqrt{1}}{2} = \dfrac{1}{2}$

b. $\lim_{x\to 0} \dfrac{(1+h)^4 - 1}{h} =$

$\lim_{x\to 0} \dfrac{1 + 4h + 6h^2 + 4h^3 + h^4 - 1}{h} =$

$\lim_{x\to 0} \dfrac{h\left(4 + 6h^1 + 4h^2 + h^3\right)}{h} =$

$\lim_{x\to 0} 4 + 6h^1 + 4h^2 + h^3 = 4$

c. $\lim_{x\to -\infty} \left(x - \sqrt{x}\right)$ DNE because the function is not defined for $x < 0$

d. $\lim_{x\to\infty} \left(x + \sqrt{x}\right) = \lim_{x\to\infty} x + \lim_{x\to\infty} \sqrt{x} = \infty + \infty = \infty$

5. Find the vertical and horizontal asymptotes for $f(x) = \left(a^{-1} + x^{-1}\right)^{-1}$, where $a > 0$.

ANS: $f(x) = \dfrac{1}{\frac{1}{a} + \frac{1}{x}}$. To find vertical asymptotes, set denominator to zero: $\dfrac{1}{a} + \dfrac{1}{x} = 0$, which implies the vertical asymptote is at $x = -a$. To find the horizontal asymptotes find the limit at infinity: $\lim_{x\to\infty} \dfrac{1}{\frac{1}{a} + \frac{1}{x}} = \dfrac{1}{\frac{1}{a}} = a$. So, the horizontal asymptote is at $y = a$.

6. Use the definition of a derivative of f at a:

a. $f(x) = x^3 - 2x$, a = 2.

ANS: $f'(x) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \dfrac{(a+h)^3 - 2(a+h) - \left(a^3 - 2a\right)}{h} =$

$\lim_{h\to 0} \dfrac{a^3 + 3a^2h + 3ah^2 + h^3 - 2a - 2h - a^3 + 2a}{h} = \lim_{h\to 0} \dfrac{3a^2h + 3ah^2 + h^3 - 2h}{h} =$

$\lim_{h\to 0} \dfrac{h\left(3a^2 + 3ah^1 + h^2 - 2\right)}{h} = \lim_{h\to 0} \dfrac{h\left(3a^2 + 3ah^1 + h^2 - 2\right)}{h} = \lim_{h\to 0}\left(3a^2 + 3ah^1 + h^2 - 2\right) = \left(3a^2 - 2\right)$

So, $f'(2) = 10$. This is the slope of the tangent line at $x = 2$.

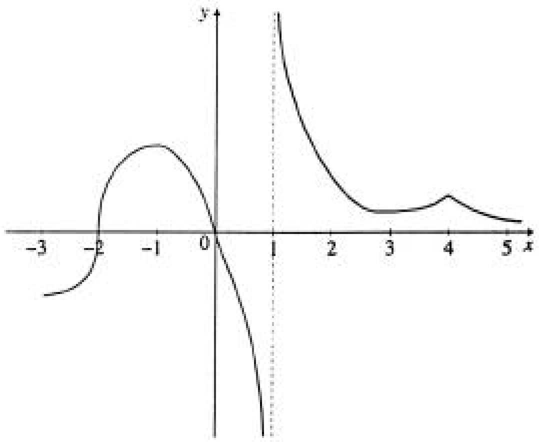
b. Find the equation of the tangent line to f at $x = 2$.

ANS: $f(2) = 4$. So, the equation of the tangent line is: $y - 4 = 10(x - 2)$.

7. If $f(x) = x - \dfrac{2}{x}$, estimate $f'(3)$ to 4 decimals. Explain.

ANS: There are several ways to estimate $f'(3)$. One way is as follows: $f(3) = 3 - \dfrac{2}{3} = \dfrac{7}{3}$ and $f(3.0001) \approx 2.33346$. So, $f'(3) = 1.2222$ is the slope of the secant line as the two points get closer:

8. The graph of g is given below.



a. For what value(s) of x is $g(x)$ not differentiable? Justify your answer(s).

ANS: $g(x)$ not differentiable at:

$x = -2$ because vertical tangent line,

$x = 1$ because $g(x)$ is not continuous,

$x = 4$ because cusp.