MAT 195 – Spring Quarter 2002
TEST 3 – Answers

NAME____

Show work and write clearly.

For #1-12, choose only 10 and find y¢. Simplify.

1. $y = \frac{e^x}{1+x^2}$

ANS: $y' = \frac{(1+x^2)e^x - 2xe^x}{\left(1+x^2\right)^2} \Rightarrow y' = \frac{(x-1)^2 e^x}{\left(1+x^2\right)^2}$

2. $y = \sec x - 2\cos x$

ANS: $y' = \sec x \tan x + 2\sin x$

3. $y = x \sin x \cos x$

ANS: Use product rule where U = $x\sin x$ and V = $\cos x$:

$y' = -x\sin^2 x + \cos x \sin x + x\cos^2 x = \cos x \sin x + x\left(\cos^2 x - \sin^2 x\right) = \cos x \sin x + x\cos(2x)$

4. $y = \sin\left(e^x\right)$

ANS: Use chain rule: $y' = e^x \cos\left(e^x\right)$

5. $y = \sin^2\left(\cos(kx)\right)$

ANS: Use chain rule:

$y' = 2\sin\left(\cos(kx)\right) \cdot \cos(\cos(kx)) \cdot (-\sin(kx)) \cdot k = -2k\sin\left(\cos(kx)\right) \cdot \cos(\cos(kx)) \cdot \sin(kx)$

6. $y = \sqrt{1 + 2\tan x}$

ANS: Use chain rule:

$y = \sqrt{1+2\tan x} = \left(1+2\tan x\right)^{1/2}$

$y' = \frac{1}{2}\left(1+2\tan x\right)^{-1/2} \cdot 2\sec^2 x = \left(1+2\tan x\right)^{-1/2}\sec^2 x$

7. $\sqrt{x+y} + \sqrt{xy} = 6$

ANS: Use implicit differentiation:

$\sqrt{x+y} + \sqrt{xy} = 6 \Rightarrow (x+y)^{1/2} + (xy)^{1/2} = 6$

So, $\frac{1}{2}(x+y)^{-1/2} \cdot (1+y') + \frac{1}{2}(xy)^{-1/2}(y + xy') = 0$

Distribute and rearrange to solve for y':

$y' = \frac{-x^{-1/2}y^{1/2} - (x+y)^{-1/2}}{(x+y)^{-1/2} + x^{1/2}y^{-1/2}}$

8. $y = \left(\tan^{-1}\sqrt{x}\right)^2$

ANS: Use chain rule:

$$y' = 2\left(\tan^{-1}\sqrt{x}\right)\cdot\frac{1}{1+\left(\sqrt{x}\right)^2}\cdot\frac{1}{2}x^{-1/2} = \tan^{-1}\sqrt{x}\cdot\frac{1}{1+x^2}\cdot x^{-1/2}$$

9. $y = x^{1/x}$

ANS: Use ln and implicit differentiation:

$$\ln y = \frac{1}{x}\ln x \Rightarrow \frac{1}{y}\cdot y' = -x^{-2}\ln x + x^{-1}\frac{1}{x} \Rightarrow y' = \frac{y}{x}(1-\ln x)$$

10. $x\cos y + y\cos x = 1$

ANS: Use implicit differentiation and product rule:

$$\cos y + x(-\sin y)y' + y'\cos x + y(-\sin x) = 0$$

$$\Rightarrow y'(\cos x - x\sin y) = -\cos y + y\sin x$$

$$\Rightarrow y' = \frac{-\cos y + y\sin x}{\cos x - x\sin y}$$

11. $y = \ln\left(\csc(5x)\right)$

ANS: Use chain rule:

$$y' = \frac{1}{\csc(5x)}\cdot(-\csc(5x)\cot(5x))\cdot 5 = -5\cot(5x)$$

12. $y = 5^{x\tan x}$

ANS: The derivative of an exponential function [$y = 5^x$] is $y' = \ln 5\cdot 5^x$.

Use the chain rule:

$$y' = \ln 5\cdot 5^{x\tan x}\cdot(\tan x + x\sec^2 x)$$

For #13-16, choose only 3 and find y¢¢.

13. $y = \cos(\ln x)$

ANS: Use chain rule:

$$y' = -\sin(\ln x)\cdot\frac{1}{x} = -\frac{\sin(\ln x)}{x}$$

Use quotient rule and chain rule:

$$y'' = \frac{-x\cos(\ln x)\cdot\frac{1}{x} - \left(-\sin(\ln x)\right)}{x^2} = \frac{\sin(\ln x) - \cos(\ln x)}{x^2}$$

14. $y = e^{-5x}\cos(3x)$

ANS: Use product rule and chain rule:

$$y' = -5e^{-5x}\cos(3x) + e^{-5x}(-\sin(3x))\cdot 3 = -5e^{-5x}\cos(3x) - 3e^{-5x}\sin(3x)$$

$$y'' = 25e^{-5x}\cos(3x) + 15e^{-5x}\sin(3x) + 15e^{-5x}\sin(3x) - 9e^{-5x}\cos(3x) = e^{-5x}(16\cos(3x) + 30\sin(3x))$$

15. $y = \dfrac{x^2 - x - 2}{x + 2}$

ANS: Use quotient rule:

$$y' = \frac{(x+2)(2x-1) - (x^2 - x - 2)(1)}{(x+2)^2} = \frac{x^2 + 4x}{(x+2)^2}$$

$$y'' = \frac{(x^2 + 4x + 4)(2x + 4) - (x^2 + 4)(2x + 4)}{(x+2)^4} = \frac{8}{(x+2)^3}$$

16. $y = e^x\sqrt{x}$

ANS: Use product rule:

$$y = e^x x^{1/2}$$

$$y' = e^x x^{1/2} + \frac{1}{2}e^x x^{-1/2}$$

$$y'' = e^x x^{1/2} + \frac{1}{2}e^x x^{-1/2} + \frac{1}{2}e^x x^{-1/2} - \frac{1}{2}\cdot\frac{1}{2}e^x x^{-3/2} = e^x x^{1/2} + e^x x^{-1/2} - \frac{1}{4}e^x x^{-3/2}$$