MAT 254 – Fall Quarter 2002
Final Exam

NAME_____

Show work and write clearly. Answers without work to support them will not receive full credit. Answers without correct notation will not receive full credit.

Answer *only* 11 of the 12 questions. Where necessary, estimate to 4 decimal places.

1. Without using the *allsums* program, estimate the area under the graph of $f(x) = x^2 + \frac{2}{x}$ from $x = 2$ to $x = 4$ using four approximating rectangles and midpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate? Explain.

2. Sketch the region enclosed by $y = x - 1$ and $x = 3 - y^2$. Sketch the area.

3. a. Find the average value of $f(x) = \frac{3x}{\sqrt{1 - x^2}}$ from $x = 0$ to $x = \frac{1}{2}$.

   b. Find c such that average value of f equals $f(c)$. Explain.

   c. Sketch the graph of the function and a rectangle whose area is the same as the area under the graph of f.

4. Find the volume of the solid formed by revolving the region bounded by $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$. Sketch the area.

5. Find the length of the curve $y = \ln(\cos x)$, $0 \le x \le \frac{p}{4}$.

6. Find the area of the surface of revolution obtained by rotating the curve $y = \frac{x^5}{10} + \frac{1}{6x^3}$, $1 \le x \le 2$ about the x-axis.

7. Solve the initial value problem: $e^{-x^2}(y^2 - 1)y' + xy = 0$, $y(0) = 1$.

8. Derive the formula for the derivative of $\sin^{-1}(x)$. Show all steps and be specific.

9. Find the derivatives of the following:

   a. $y = \sin^{-1}(x) + x\sqrt{1 - x^2}$ b. $y = (\sin x)^{\tan x}$

10. Find the following limits:

    a. $\lim_{x \to -\infty} \frac{x^2}{e^{-x}}$ b. $\lim_{x \to \infty} e^{-x}\sqrt{x}$ c. $\lim_{x \to \infty} \left(x - \sqrt{x^2 + x}\right)$

11. Find $\int \frac{\tan^3 x}{\cos^4 x}\,dx$

12. Find $\int e^x \cos(2x)\,dx$