

**MAT 254 – Fall Quarter 2002**  
**Test 4**

NAME \_\_\_\_\_

**Show work and write clearly.**

1. (10 pts.) Derive the formula for the derivative of  $\cos^{-1}(x)$ .
2. (30 pts.) Find the derivatives of the following:
  - a.  $y = \cos^{-1}(\sin x)$
  - b.  $y = \sec^{-1}(e^x)$
  - c.  $y = x^3 \sqrt{1 + x^2}$
  - d.  $y = (\ln x)^{\tan x}$
  - e.  $y = x^{(e^x)}$
3. (10 pts.) One hundred fruit flies are placed in a breeding container that can support a population of at most 5000 flies. If the population grows with a constant relative growth rate of 2% per day, how long will it take for the container to reach capacity?
4. (40 pts.) Find the following limits:
  - a.  $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 5} - 3}{x^2 - 4}$
  - b.  $\lim_{x \rightarrow 0^+} x \ln x$
  - c.  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x + x^2}$
  - d.  $\lim_{x \rightarrow \infty} \left( x - \sqrt{x^2 + x} \right)$
  - e.  $\lim_{x \rightarrow 0} \frac{\sqrt{1 + x} - 1 - x/2}{x^2}$
5. (10 pts.) Forty percent of a radioactive substance decays in 5 years. How long would it take the sample to decay to 1% of its original amount?