

**MAT 254 – Winter Quarter 2003**  
**Final Exam**

NAME \_\_\_\_\_

**Show work and write clearly. Answers without work to support them will not receive full credit. Answers without correct notation will not receive full credit.**

**Where necessary, estimate to 4 decimal places.**

1. (20 pts.). Without using the *allsums* program, estimate the area under the graph of  $f(x) = (1 - x)(2x + 3)$  from  $x = -3$  to  $x = 3$  using three approximating rectangles and midpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate? Explain.

---

2. (15 pts.). Sketch the region enclosed by  $y = x^2$ ,  $y = 8 - x^2$  and  $4x - y + 12 = 0$ . Find the area.

---

3. (20 pts.). Find the volume of the solid formed by revolving the region bounded by  $y = \sin x$ ,  $x = 2\pi$ ,  $x = 3\pi$  and  $y = 0$  about the  $y$ -axis. Sketch the area.

---

4. (5 pts.). Use Simpson's rule with  $n = 30$  to find the length of the curve  $y = \csc(2x)$ ,  $0.5 \leq x \leq 1.5$ .

---

5. (10 pts.). Derive the formula for the derivative of  $\sin^{-1}(x)$ . Show all steps and be specific.

---

6. (15 pts.). Find  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ .

---

7. (10 pts.). Find  $\int_0^2 x^2 3^x dx$ .

---

(5 pts.) **Answer only ONE of the following:**

8. Find  $\int_e^\infty \frac{1}{x \ln x} dx$

9. Find  $\int \cos^4 x \sin^3 x dx$ .