MAT 254 – Winter Quarter 2003
Test 1 – Answers

NAME____

Show work and write clearly.

1. (10 pts.) Derive the formula for the derivative of $\sin^{-1}(x)$. **ANS:** See page 229 of your text.

2. (20 pts.) Find the antiderivatives:

a. $f(x) = \sqrt[3]{x^2} + \sec x \tan x + \frac{5}{x} - p$ **ANS:** Rewrite as $f(x) = x^{2/3} + \sec x \tan x + \frac{5}{x} - p$.
The antiderivative is $F(x) = \frac{x^{5/3}}{5/3} + \sec x + 5\ln|x| - px + C$.

b. $k(x) = \frac{1}{x^3} + \frac{1}{x\sqrt{x}}$ **ANS:** Rewrite as $k(x) = x^{-3} + x^{-3/2}$. The antiderivative is
$K(x) = \frac{x^{-2}}{-2} + \frac{x^{-1/2}}{-1/2} + C = -\frac{x^{-2}}{2} - 2x^{-1/2} + C$.

3. (10 pts.) Find the antiderivative: $f''(x) = \sin x,\ f'(0) = 1,\ f(0) = 6$
ANS: First find $f'(x)$: $f'(x) = -\cos x + C$. Now use the condition $f'(0) = 1$ to solve for C:
$1 = -\cos(0) + C \Rightarrow C = 2$. So, $f'(x) = -\cos x + 2$. Now find $f(x)$: $f(x) = -\sin x + 2x + C$. Now use the condition $f(0) = 6$ to solve for C: $6 = -\sin(0) + 2(0) + C \Rightarrow C = 6$. Thus,
$f(x) = -\sin x + 2x + 6$.

4. (30 pts.) Find the derivatives:

a. $g(t) = \csc^{-1}(t^2)$ **ANS:** $g'(t) = -\frac{1}{t^2\sqrt{(t^2)^2 - 1}} \cdot 2t = -\frac{2}{t\sqrt{t^4 - 1}}$

b. $h(x) = \ln(1 - e^{-x})^{2x}$ **ANS:** Use exponent property of logs: $h(x) = 2x \cdot \ln(1 - e^{-x})$. Now use product and chain rules: $h'(x) = 2 \cdot \ln(1 - e^{-x}) + 2x \cdot \frac{1}{1 - e^{-x}} \cdot -e^{-x} \cdot -1 = 2\ln(1 - e^{-x}) + \frac{2xe^{-x}}{1 - e^{-x}}$.

c. $j(x) = (\sin x)^{\cos x}$ **ANS:** Use logarithmic differentiation: $\ln y = \cos x \cdot \ln(\sin x)$
$\Rightarrow \frac{1}{y} \cdot y' = \cos x \cdot \frac{1}{\sin x} \cdot \cos x - \sin x \ln(\sin x) \Rightarrow y' = (\sin x)^{\cos x}\left[\frac{\cos^2 x}{\sin x} - \sin x \ln(\sin x)\right]$

d. $y \sin x = x \cos y$ **ANS:** Use implicit differentiation:
$y' \sin x + y \cos x = \cos y + x \cdot -\sin y \cdot y'$. Now solve for y´:
$y' \sin x + x \sin y \cdot y' = \cos y - y \cos x \Rightarrow y'(\sin x + x \sin y) = \cos y - y \cos x$
$\Rightarrow y' = \dfrac{\cos y - y \cos x}{\sin x + x \sin y}$

5. (30 pts.) Find the following limits:

a. $\lim\limits_{x \to \infty} \dfrac{\ln(\ln x)}{\ln x}$ **ANS:** This is type ∞/∞, so use l'Hôpital's Rule: $\lim\limits_{x \to \infty} \dfrac{\ln(\ln x)}{\ln x} \overset{H}{=} \lim\limits_{x \to \infty} \dfrac{\left(\dfrac{1}{x \ln x}\right)}{\left(\dfrac{1}{x}\right)}$
$\Rightarrow \lim\limits_{x \to \infty} \dfrac{x}{x \ln x} = \lim\limits_{x \to \infty} \dfrac{1}{\ln x} = 0$.

b. $\lim\limits_{x \to 0} \dfrac{\sin(ax)}{\sin(bx)}$ **ANS:** This is type 0/0, so use l'Hôpital's Rule:
$\lim\limits_{x \to 0} \dfrac{\sin(ax)}{\sin(bx)} \overset{H}{=} \lim\limits_{x \to 0} \dfrac{a \cos(ax)}{b \cos(bx)} = \dfrac{a}{b}$.

c. $\lim\limits_{x \to 2} \left(\dfrac{8}{x^2 - 4} - \dfrac{x}{x - 2} \right)$ **ANS:** This is type ∞ – ∞ so first add rational expressions:
$\lim\limits_{x \to 2} \left(\dfrac{8}{x^2 - 4} - \dfrac{x}{x - 2} \cdot \dfrac{x + 2}{x + 2} \right) = \lim\limits_{x \to 2} \left(\dfrac{8 - x^2 - 2x}{x^2 - 4} \right)$. This is type 0/0, so use l'Hôpital's Rule:
$\lim\limits_{x \to 2} \left(\dfrac{8 - x^2 - 2x}{x^2 - 4} \right) \overset{H}{=} \lim\limits_{x \to 2} \dfrac{-2x - 2}{2x} = -\dfrac{3}{2}$.

d. $\lim\limits_{x \to 3} \dfrac{x - 3}{\ln(2x - 5)}$ **ANS:** This is type 0/0, so use l'Hôpital's Rule:
$\lim\limits_{x \to 3} \dfrac{x - 3}{\ln(2x - 5)} \overset{H}{=} \lim\limits_{x \to 3} \dfrac{1}{\left(\dfrac{1}{2x - 5}\right) \cdot 2} = \lim\limits_{x \to 3} \left(\dfrac{2x - 5}{2} \right) = \dfrac{1}{2}$.