MAT 254 – Winter Quarter 2002
Test 2 – Answers

NAME____

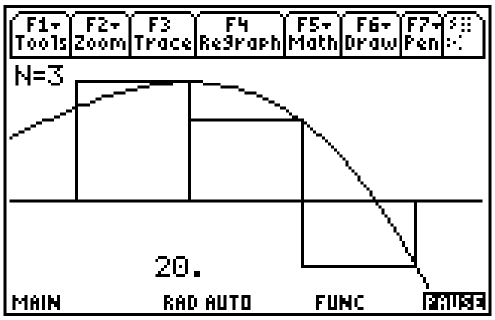
Show work and write clearly.

1. (30 pts.) <u>Without</u> using the *allsums* program,

(a). Estimate the area (to 4 decimal places) under the graph of $f(x) = 3x + 4x^2 - x^3$ from $x = 2$ to $x = 5$ using three approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate? Explain.

(b). Repeat using midpoints.

ANS:

(a). 

$\Delta x = \dfrac{b-a}{n} = \dfrac{6-3}{3} = 1$ is the width of the approximating rectangles.

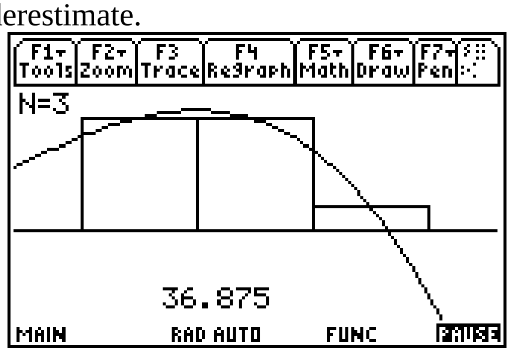
$R_3 = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x = f(3)\Delta x + f(4)\Delta x + f(5)\Delta x$.

$= \left(3(3) + 4(3)^2 - (3)^3\right)(1) + \left(3(4) + 4(4)^2 - (4)^3\right)(1) + \left(3(5) + 4(5)^2 - (5)^3\right)(1)$

$= (9 + 36 - 27)(1) + (12 + 64 - 64)(1) + (15 + 100 - 125)(1)$

$= (18)(1) + (12)(1) + (-10)(1) = 20$.

Since the function decreases more rapidly on (3, 5) than it increases on (2, 3), the RHS is an underestimate.

(b). 

$\Delta x_i = \dfrac{b-a}{n} = \dfrac{5-2}{3} = 1$ is the width of the approximating rectangles.

$M_3 = f\left(\dfrac{x_0 + x_1}{2}\right)\Delta x + f\left(\dfrac{x_1 + x_2}{2}\right)\Delta x + f\left(\dfrac{x_2 + x_3}{2}\right)\Delta x$

$= f(2.5)\Delta x + f(3.5)\Delta x + f(4.5)\Delta x$

$= \left(3(2.5) + 4(2.5)^2 - (2.5)^3\right)(1) + \left(3(3.5) + 4(3.5)^2 - (3.5)^3\right)(1) + \left(3(4.5) + 4(4.5)^2 - (4.5)^3\right)(1)$

$= (7.5 + 25 - 15.625)(1) + (10.5 + 49 - 42.875)(1) + (13.5 + 81 - 91.125)(1)$

$= (16.875)(1) + (16.625)(1) + (3.375)(1) = 36.875 = \dfrac{295}{8}$.

Since the function is concave down on (2, 5), the MIDPT is an overestimate.

2. (40 pts.) Use the Fundamental Theorem of Calculus to evaluate the integral, or explain why it does not exist.

a. $\int_{p}^{3p} (x + \sin x)dx$ **ANS:** $\left(\frac{x^2}{2} - \cos x\right)\Big|_{x=p}^{3p} = \frac{(3p)^2}{2} + \cos(3p) - \left(\frac{(p)^2}{2} - \cos(p)\right)$

$$= \frac{9p^2}{2} + 1 - \left(\frac{p^2}{2} + 1\right) = 4p^2 \approx 39.4784$$

b. $\int_{0}^{2} e^{2x}dx$ **ANS:** $\frac{e^{2x}}{2}\Big|_{x=0}^{2} = \frac{e^{2(2)}}{2} - \frac{e^{2(0)}}{2} = \frac{e^4}{2} - \frac{e^0}{2} = \frac{e^4}{2} - \frac{1}{2} \approx 26.7991$

c. $\int_{1}^{3} \frac{2}{\sqrt[3]{x^2}}dx$ **ANS:** $\frac{2x^{1/3}}{1/3}\Big|_{x=1}^{3} = 6(3)^{1/3} - 6(1)^{1/3} = 6(3)^{1/3} - 6 \approx 2.6535$

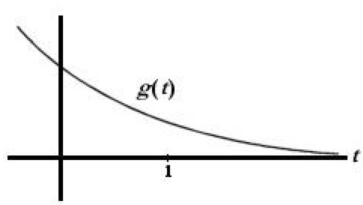
3. (10 pts.) Calculate the left-hand, right-hand, midpoint and trapezoid sums with 100 subdivisions. Which of these sums are overestimates and which are underestimates? Explain. Estimate the value of the definite integral. Explain. $\int_{0}^{3} \left[\ln\left(30 - x^3\right) - 2\right]dx$.

ANS: Using allsums: L_{100} = 3.20645; R_{100} = 3.13738; M_{100} = 3.17292; T_{100} = 3.17192.
The function is decreasing on (0, 3). Since the function is decreasing, the R_{100} is an underestimate and the L_{100} is an overestimate. Since the curve is concave down on (0, 3), T_{100} is an underestimate and M_{100} is an overestimate. Finally, there are various answers for the estimate of the value of the definite integral – it must be between M_{100} and T_{100}.

4. (10 pts.) The graph of g is shown below. The results from the left, right, midpoint and trapezoid rules used to approximate $\int_{0}^{1} g(t)dt$, with the same number of subdivisions for each rule, are as follows: 0.601, 0.632, 0.633, 0.664.

a. Match each rule with its approximation. Explain.

b. Between which two approximations does the true value of the integral lie? Explain.



ANS: a. LHS = 0.664; RHS = 0.601; MIDPT = 0.632; TRAP = 0.633. The function is decreasing on [0, 1], so the LHS is an overestimate and the RHS is an underestimate. Thus, the RHS needs to be the largest value and the LHS needs to be the smallest value (0.664 > 0.601). The function is concave up, so the TRAP is an overestimate and the MIDPT is an underestimate.

5. (10 pts.) <u>Without</u> using the *allsums* program, is $\int_{-1}^{1} e^{x^2}\,dx$ positive, negative or zero? Explain.

ANS: The area between the curve and the *x*-axis between $x = -1$ and $x = 1$ is above the *x*-axis, so the definite integral is positive.