MAT 254 – Winter Quarter 2002
Test 3

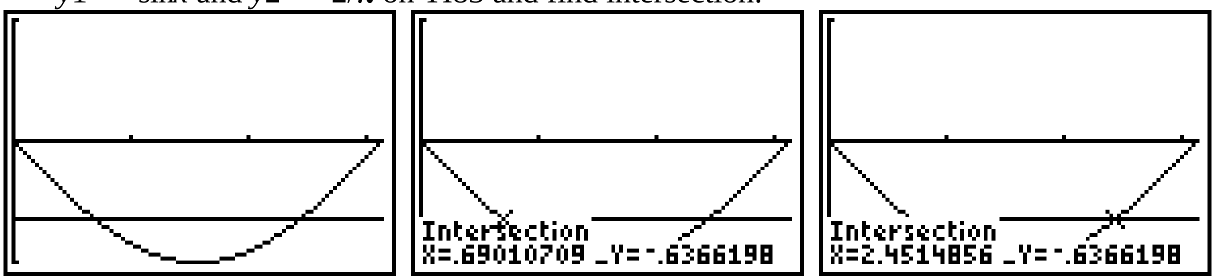
NAME____
Show work and write clearly.

1. (30 pts.)

a. Find the average value of $f(x) = -\sin x$ on $[0, \pi]$.

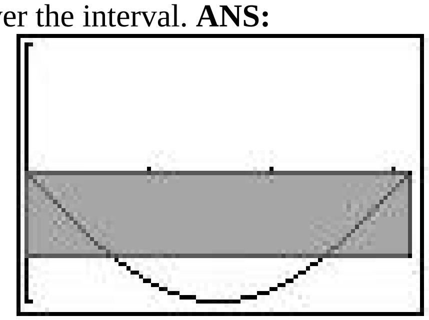
ANS: $f_{ave} = \dfrac{1}{p - 0}\int_0^p -\sin x dx = \dfrac{1}{p}\cos x\Big|_0^p = \dfrac{-1-1}{p} = \dfrac{-2}{p}$.

b. Find the value c such that $f(c) = f_{\text{ave}}$.
ANS: That is, find c such that $f(c) = -\sin c = -2/p$. One method is to graph $y1 = -\sin x$ and $y2 = -2/\pi$ on TI83 and find intersection:



Thus, there are two possible values of c: $c \approx 0.6901$ and $c \approx 2.4515$.

c. Sketch the graph of $f(x)$ and construct a rectangle over the interval whose area is the same as the area under the graph of $f(x)$ over the interval. **ANS:**



2. (48 pts.) Find the following integrals:

a. $\int \dfrac{y}{\sqrt{y+1}}dy$ **ANS:** let $u = y + 1$, then $du = dy$ and $y = u - 1$. So, $\int \dfrac{y}{\sqrt{y+1}}dy =$

$$\int \frac{u-1}{\sqrt{u}}du = \int \frac{u}{\sqrt{u}}du - \int \frac{1}{\sqrt{u}}du = \int u^{1/2}du - \int u^{-1/2}du$$

$$= \frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2} + C = \frac{2(y+1)^{3/2}}{3} - 2(y+1)^{1/2} + C$$

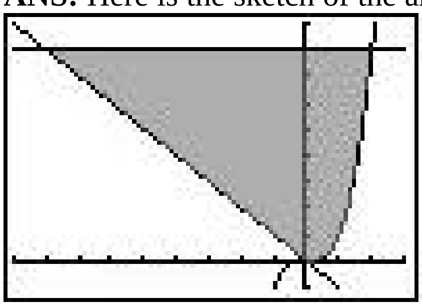
b. $\int \tan^3(5x)\sec^2(5x)dx$ **ANS:** let $u = \tan(5x)$, then $du = 5\sec^2(5x)dx$. So,

$$\int \tan^3(5x)\sec^2(5x)dx = \frac{1}{5}\int u^3 du = \frac{1}{5}\frac{u^4}{4} + C = \frac{1}{20}\tan^4(5x) + C$$

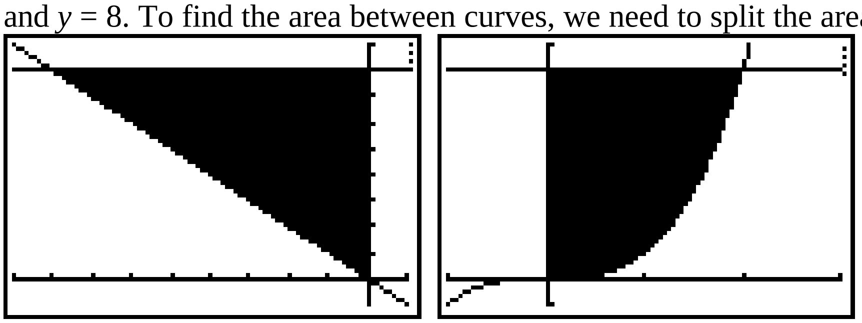
c. $\int_1^{3/2} \left[\csc^2(\cos(3t))\right]\sin(3t)dt$ **ANS:** let $u = \cos(3t)$, then $du = -3\sin(3t)dt$. When $t = 1$, $u = -0.99$ and when $t = 1.5$, $u = -0.21$. So, $\int_1^{3/2} \left[\csc^2(\cos(3t))\right]\sin(3t)dt \approx -\frac{1}{3}\int_{-0.99}^{-0.21}\csc^2(u)du = \frac{1}{3}\cot u\Big|_{-0.99}^{-0.21} \cong$ $[-4.7 - (-0.66)]/3 = -1.35$.

d. $\int_{-\sqrt{2}}^{0} x(2 - x^2)^3 dx$ **ANS:** let $u = 2 - x^2$, then $du = -2xdx$. When $x = -\sqrt{2}$, $u = 0$ and when $x = 0$, $u = 2$. So, $\int_0^2 x(2 - x^2)^3 dx = -\frac{1}{2}\int_0^2 u^3 du = -\frac{1}{2}\frac{u^4}{4}\Big|_0^2 = -2$.

3. (22 pts.) Sketch the area between $y = x^3$, $y = -x$, $y = 8$. Find the area.
ANS: Here is the sketch of the area:



$a = -8$ is the intersection point between $y = -x$ and $y = 8$; $b = 2$ is the intersection point between $y = x^3$ and $y = 8$. To find the area between curves, we need to split the area into two parts:



$$\text{Area} = \int_{-8}^{0}(8 - (-x))dx + \int_0^2 (8 - x^3)dx = \left(8x + \frac{x^2}{2}\right)\Bigg|_{-8}^{0} + \left(8x - \frac{x^4}{4}\right)\Bigg|_0^2$$

$$= 0 - \left[8(-8) + \frac{(-8)^2}{2}\right] + 8(2) - \frac{2^4}{4} - 0 = 32 + 16 - 4 = 44$$