MAT 254 – Winter Quarter 2002
Test 4 – Answers

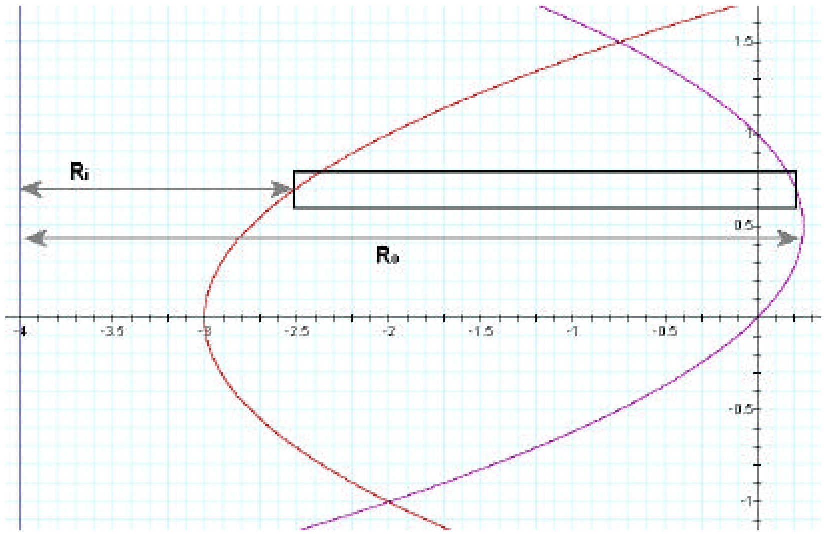
NAME____

Show work and write clearly.

Make sure graphs show approximating rectangle, *R*(*x*), *r*(*x*), *p*(*x*) and *h*(*x*).

1. (30 pts.) Use the disk method to find the volume of the solid obtained by rotating the region bounded by $x = y - y^2$ and $x = y^2 - 3$ about $x = -4$.

ANS:



The purple function is $x = y - y^2$ and the red function is $x = y^2 - 3$. To find the intersection points set the functions equal to one another, $y - y^2 = y^2 - 3$ and use the quadratic formula: y = –1, 3/2.

The outer radius, R_o or $R(y)$: the function furthest to right minus the axis of rotation, i.e.,

$y - y^2 - (-4) = y - y^2 + 4$.

The inner radius, R_i or $r(y)$: the function to left minus the axis of rotation, i.e., $y^2 - 3 - (-4) = y^2 + 1$.

So, V $= p\int_{-1}^{3/2}\left([R(y)]^2 - [r(y)]^2\right)dy = p\int_{-1}^{3/2}\left([y - y^2 - 4]^2 - [y^2 + 1]^2\right)dy$. Be careful to distribute fully the functions when squaring them:

$$V = p\int_{-1}^{3/2}\left(-7y^2 - 2y^3 + 8y + y^4 + 16 - (y^4 + 2y^2 + 1)\right)dy$$

$$= p\int_{-1}^{3/2}\left(-2y^3 - 9y^2 + 8y + 15\right)dy = p\left(-\frac{y^4}{2} - 3y^3 + 4y^2 + 15y\right)\Bigg|_{-1}^{3/2} = \frac{875}{32}p$$

2. (30 pts.) Find the following integrals:

a. $\int \left(\tan^{-1} x\right) dx$

ANS:

U	**dV**
$\tan^{-1} x$	1
$\dfrac{1}{1+x^2}$	x

So, $\int \left(\tan^{-1} x\right) dx = x\tan^{-1} x - \int \frac{x}{1+x^2} dx$. Now use substitution, let $t = 1 + x^2$, $dt = 2xdx$. So,

$$\int \left(\tan^{-1} x\right) dx = x\tan^{-1} x - \frac{1}{2}\int \frac{1}{t} dt = x\tan^{-1} x - \frac{1}{2}\ln|t| + C = x\tan^{-1} x - \frac{1}{2}\ln\left(1+x^2\right) + C .$$

b. $\int \left(x^4 \ln x\right) dx$

ANS:

U	**dV**
$\ln x$	x^4
$\dfrac{1}{x}$	$(1/5)\, x^5$

So, $\int \left(x^4 \ln x\right) dx = \frac{1}{5}x^5 \ln x - \frac{1}{5}\int \frac{x^5}{x} dx = \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4 dx = \frac{1}{5}x^5 \ln x - \frac{1}{5}\left(\frac{1}{5}\right)x^5 + C$.

c. $\int_0^1 \left(x^3 e^{x^2}\right) dx$

ANS: Let $t = x^2$, $dt = 2xdx$.

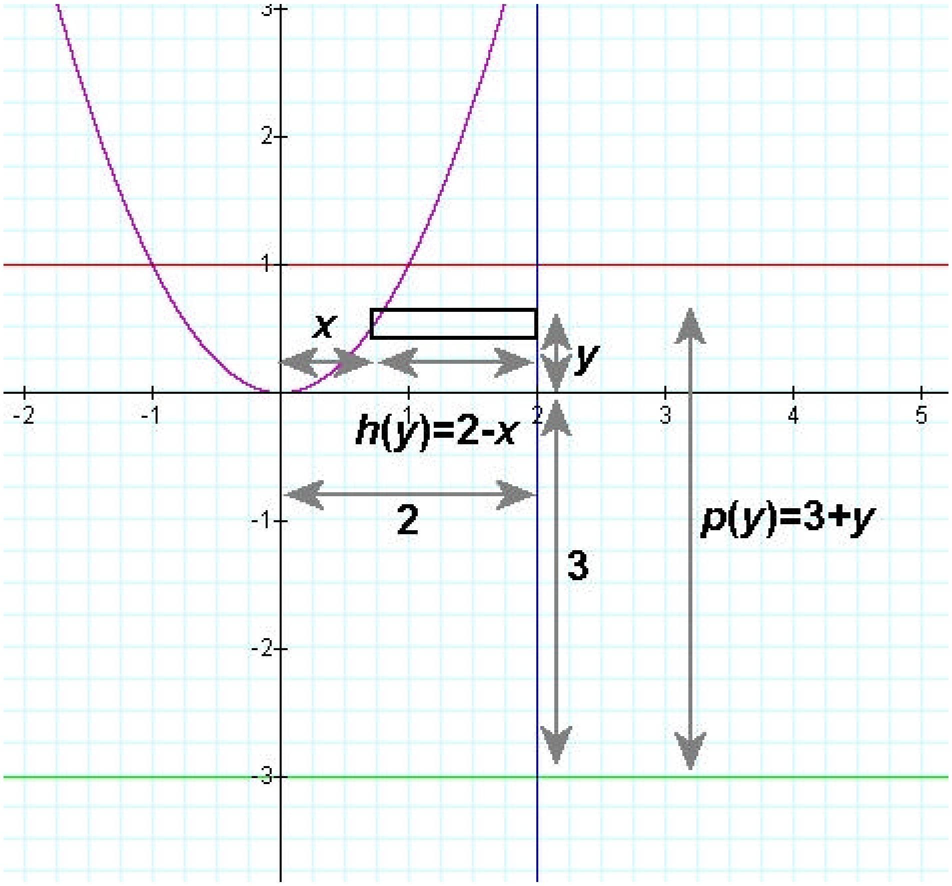
So, $\int_0^1 \left(x^3 e^{x^2}\right) dx = \frac{1}{2}\int_0^1 \left(x^2 e^t\right) dt = \frac{1}{2}\int_0^1 \left(te^t\right) dt$.

U	**dV**
t	e^t
1	e^t

So, $\int_0^1 \left(x^3 e^{x^2}\right) dx = \frac{1}{2}\int_0^1 \left(te^t\right) dt = \frac{1}{2}\left[te^t - e^t\right]_0^1 = \frac{1}{2}$.

3. (30 pts.) Use the shell method to find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 1$ and $x = 2$ about $y = -3$.

ANS:



$h(y) = 2 - x = 2 - \sqrt{y}$. So, V = $2p\int_0^1 (p(y)h(y))dy = 2p\int_0^1 (2-\sqrt{y})(y+3)dy$

$= 2p\int_0^1 (6 + 2y - 3y^{1/2} - y^{3/2})dy = 2p\left(6y + y^2 - 2y^{3/2} - \frac{2}{5}y^{5/2}\right)\Big|_0^1$

$= 2p\left(6 + 1 - 2 - \frac{2}{5}\right) - 0 = \frac{46}{5}p$.

4. (10 pts.) Solve: $y' = \frac{xy}{3\ln y}$.

ANS: $\frac{dy}{dx} = \frac{xy}{3\ln y} \Rightarrow \frac{\ln y}{y}dy = \frac{1}{3}xdx \Rightarrow \int \frac{\ln y}{y}dy = \frac{1}{3}\int xdx$.. To find the integral on the left hand side, use substitution, let $u = \ln y$, $du = dy/y$. So,

$\int \frac{\ln y}{y}dy = \frac{1}{3}\int xdx \Rightarrow \int udu = \frac{1}{3}\int xdx \Rightarrow \frac{u^2}{2} = \frac{1}{3}\frac{x^2}{2} + C \Rightarrow u^2 = (\ln y)^2 = \frac{1}{3}x^2 + C$

$\Rightarrow \ln y = \sqrt{\frac{1}{3}x^2 + C} \Rightarrow e^{\ln y} = e^{\sqrt{\frac{1}{3}x^2 + C}} \Rightarrow y = e^{\sqrt{\frac{1}{3}x^2 + C}}$.