MAT 195 – Fall Quarter 2002
TEST 1

NAME____
Show work and write clearly.

1. *15 pts.* Let $h(x) = \sqrt{x^3 + 9}$. Find $h^{-1}(x)$. State the domain and range for $h(x)$ and $h^{-1}(x)$.

ANS: To find the inverse change $h(x)$ to y, switch x and y, then solve for y. The result is $h^{-1}(x) = \sqrt[3]{x^2 - 9}$. The domain of $h(x)$ is restricted due to the square root. To find the domain, set the argument $x^3 + 9 \geq 0$ and solve for x to get $x \geq -\sqrt[3]{9}$ or $[-\sqrt[3]{9}, \infty)$. The domain of the function is the range of the inverse. The range of the square root function is $[0, \infty)$ and this is the domain of the inverse.

	$f(x)$	$f^{-1}(x)$
Domain	$[-\sqrt[3]{9}, \infty)$	$[0, \infty)$
Range	$[0, \infty)$	$[-\sqrt[3]{9}, \infty)$

2. *10 pts.* Let $f(x) = \sqrt{(\tan(-5x))^3}$. Write f as a composition of five functions.
ANS: Choose $g(x) = \sqrt{x}$, $h(x) = \tan x$, $j(x) = x^3$, $k(x) = -x$, $l(x) = 5x$. Thus, $f(x) = g(j(h(k(l(x)))))$.

3. *15 pts.* Find the domain and range of:
(a). $f(x) = \dfrac{1}{\sqrt{x - x^3}}$ (b). $g(x) = \sqrt{4 - 3x^2}$
ANS: a. The domain of $f(x)$ is restricted due to the square root. To find the domain, set the argument $x - x^3 \geq 0$ and solve for x. In order to solve a polynomial inequality you need to find critical values by setting the argument $x - x^3 = 0$. The critical values can be found by factoring and are $x = 0, 1, -1$. Graph this on a number line and use test values to check the sections of the number line. Note that the endpoints are not in the domain because they make the denominator 0. The range of the square root function is $(0, \infty)$ and this is the domain of the inverse.

b. The domain of $g(x)$ is found in an analogous manner. Note that the range of $g(x)$ can be found by looking at a graph of the function or by finding the domain of its inverse.

	$f(x)$		$g(x)$
Domain	$(-\infty,-1) \cup (0, 1)$		$\left[-\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}\right]$
Range	$(0, \infty)$		$[0, 2]$

4. *10 pts.* Determine whether *f* is even, odd or neither even nor odd. **Explain**.

(a). $f(x) = 1 + \sin x$ (b). $f(x) = x^7 - x^3$

ANS: To determine if a function is symmetric about the *y*-axis (i.e., even function), replace *x* with –*x* and note if you can simplify the equation back into the original. To determine if a function is symmetric about the origin (i.e., odd function), replace *x* with –*x* and *y* with –*y* and note if you can simplify the equation back into the original. (Another method is to determine if $f(-x) = -f(x)$. If you cannot simplify the function back into the original, then the function does not have that symmetry.

a. $f(x) = 1 + \sin x$ is neither even nor odd.

b. After replacing *x* with –*x* and *y* with –*y*, we have $-y = (-x)^7 - (-x)^3$ and this simplifies to $y = x^7 - x^3$. Thus, the function is odd.

5. *15 pts.* A small-appliance manufacturer finds that it costs $9000 to produce 1000 toaster ovens a week and $12,000 to produce 1500 toaster ovens a week.

(a). Express the cost as a function of the number of toaster oven produced, assuming that it is linear. Sketch the graph.

ANS: We are told it is linear, so we have to find the line that passes through the points (1000, 9000) and (1500, 12000). The slope of the line is $m = \dfrac{12000 - 9000}{1500 - 1000} = 6$. So the equation of the line is $y - 9000 = 6(x - 1000) \Rightarrow C(x) = y = 6x + 3000$.

(b). What is the slope and what does it represent?

ANS: The slope is 6 and it represents the cost of each additional toaster.

(c). What is the *y*-intercept of the graph and what does it represent?

ANS: The *y*-int is 3000 and it represents the fixed costs for the manufacturer.

6. *15 pts.* Suppose that $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4 - x^2}$.

ANS: The domain of *f*(*x*) is [0, ∞) and the domain of *g*(*x*) is [–2, 2] (see question #3 for the method of finding the domains).

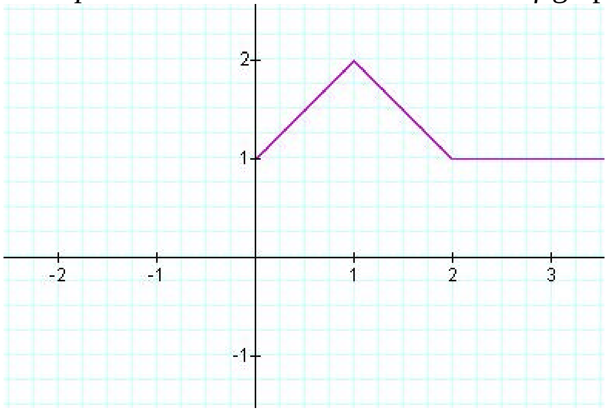
(a). What are the domains of *f* + *g*, *f*•*g*, *f*/*g*, *g*/*f*?

ANS: The domains of *f* + *g*, *f*•*g*, is the intersection of the domains of *f*(*x*) and *g*(*x*), i.e., [0, ∞) ∩ [–2, 2] = [0, 2]. The domain of *f*/*g* is the intersection of the domains of *f*(*x*) and *g*(*x*) with *g*(*x*) ≠ 0, i.e., [0, ∞) ∩ [–2, 2) = [0, 2). The domain of *g*/*f* is the intersection of the domains of *f*(*x*) and *g*(*x*) with *f*(*x*) ≠ 0, i.e., (0, ∞) ∩ [–2, 2] = (0, 2].

(b). What are *f*(*g*(*x*) and *g*(*f*(*x*))?

ANS: $f(g(x)) = f(\sqrt{4 - x^2}) = \sqrt{\sqrt{4 - x^2}} = \sqrt[4]{4 - x^2}$. $g(f(x)) = g(\sqrt{x}) = \sqrt{4 - (\sqrt{x})^2} = \sqrt{4 - x}$.

7. *10 pts.* Find a formula for the function f graphed below.



ANS: $f(x) = \begin{cases} x+1 & 0 \le x \le 1 \\ 3-x & 1 < x \le 2 \\ 1 & 2 < x \end{cases}$

8. *10 pts.* Starting with the graph of $y = x - x\ln|x|$, find the equation of the graph that results from

(a). shifting 3 units upward **ANS:** $y = x - x\ln|x| + 3$

(b). shifting 5 units to the left **ANS:** $y = (x+5) - (x+5)\ln|(x+5)|$

(c). reflecting about the x-axis **ANS:** $y = -(x - x\ln|x|)$

(d). reflecting about the y-axis **ANS:** $y = (-x) - (-x)\ln|-x| \;=\; y = -(x - x\ln|x|)$

(e). shifting 5 units to the right and then reflecting over the y-axis

ANS: $y = (-x-5) - (-x-5)\ln|-x-5| \Rightarrow y = -x - 5 + (x+5)\ln|-x-5|$