

**MAT 195 – Spring Quarter 2002**  
**TEST 2**

NAME \_\_\_\_\_

Show work and write clearly.

1. (10 points) Given  $\lim_{x \rightarrow c} f(x) = -2$  and  $\lim_{x \rightarrow c} g(x) = \frac{3}{2}$ , evaluate the following limits:

a.  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$

b.  $\lim_{x \rightarrow c} [f(x) \cdot g(x)]$

c.  $\lim_{x \rightarrow c} [4g(x) + 3f(x)]$

d.  $\lim_{x \rightarrow c} [x \cdot f(x)]$

e.  $\lim_{x \rightarrow c} \sqrt{f(x) + g(x)}$

f.  $\lim_{x \rightarrow c} [g(x) - f(x)]^2$

2. (20 points)

a. Find all points of discontinuity for the following functions. Explain.

b. For each point of discontinuity, provide the type of discontinuity. Explain.

i.  $f(x) = \begin{cases} 1 & x > 1 \\ 0 & x = 1 \\ 1 & x < 1 \end{cases}$

ii.  $f(x) = \frac{x}{x^2 - 1}$

iii.  $f(x) = \frac{x^2 - 3}{|x^2 - 3|}$

iv.  $f(x) = \frac{x - 4}{x^2 - 16}$

3. (10 points) Find a value for the constant  $k$ , if possible, that will make  $f$  continuous.

$$f(x) = \begin{cases} 7x - 2 & x \leq 1 \\ kx^2 & x > 1 \end{cases}$$

4. (20 points) Find the limits, algebraically, if they exist. If the limit does not exist, explain.

a.  $\lim_{t \rightarrow 1} \frac{t^3 - t^2 + 2t - 2}{t^2 - 3t + 2}$

b.  $\lim_{x \rightarrow 4} \frac{3 - x}{x^2 - 2x - 8}$

c.  $\lim_{x \rightarrow 1^+} \frac{x^4 - 1}{x - 1}$

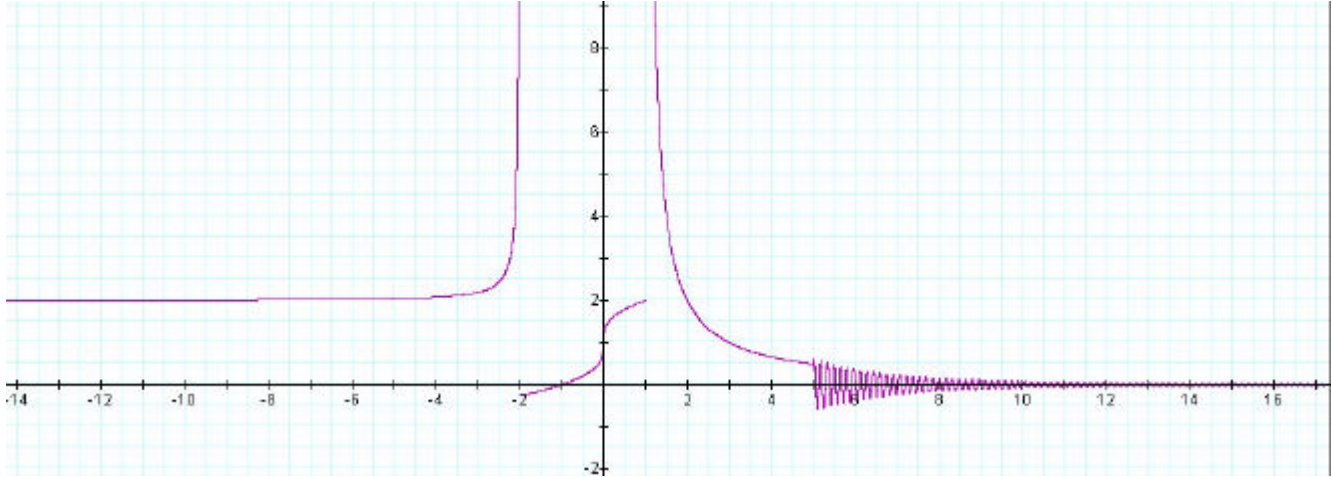
d.  $\lim_{x \rightarrow 5} \sqrt{x^3 - 3x - 1}$

e.  $\lim_{y \rightarrow 9} \frac{4 - y}{2 - \sqrt{y}}$

f.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$

5. (10 points) Use the Intermediate Value Theorem to show that the polynomial function  $f(x) = x^3 + 2x - 1$  has a root in the interval  $(0, 1)$ . Be specific on the use of the theorem.

6. (10 points) Find the equations of the asymptotes of the function graphed below. Explain the answer in terms of limits.



7. (10 points) Let

$$h(x) = \begin{cases} 2x - x^2 & 0 \leq x \leq 2 \\ 2 - x & 2 < x \leq 3 \\ x - 4 & 3 < x < 4 \\ \mathbf{p} & x \geq 4 \end{cases}$$

For each of the following numbers 2, 3 and 4, determine whether  $h$  is continuous at the number, continuous from the right or continuous from the left. Explain.

8. (10 points) The displacement (in feet) of a certain particle moving in a straight line is given by  $s = 2t^3 - 5t$ , where  $t$  is measured in seconds.

- Find the average velocity from  $t = 2$  to  $t = 5$ .
- Estimate the instantaneous velocity at  $t = 2$ .