MAT 195 – Spring Quarter 2002
TEST 2

NAME____
Show work and write clearly.

1. (10 points) Given $\lim_{x \to c} f(x) = -2$ and $\lim_{x \to c} g(x) = \frac{3}{2}$, evaluate the following limits:

ANS:

a. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{-2}{3/2} = -\frac{4}{3}$

b. $\lim_{x \to c} [f(x) \cdot g(x)] = -2\frac{3}{2} = -3$

c. $\lim_{x \to c} [4g(x) + 3f(x)] = 4\frac{3}{2} + 3(-2) = 0$

d. $\lim_{x \to c} [x \cdot f(x)] = c(-2) = -2c$

e. $\lim_{x \to c} \sqrt{f(x) + g(x)}$
DNE because negative sign in square root

f. $\lim_{x \to c} [g(x) - f(x)]^2 = \left[\frac{3}{2} - (-2)\right]^2 = \frac{49}{4}$

2. (20 points)
   a. Find all points of discontinuity for the following functions. Explain.
   b. For each point of discontinuity, provide the type of discontinuity. Explain.

ANS:

i. $f(x) = \begin{cases} 1 & x > 1 \\ 0 & x = 1 \\ 1 & x < 1 \end{cases}$

removable discontinuity at $x = 0$
since $\lim_{x \to 0} f(x) \neq f(x)$

ii. $f(x) = \frac{x}{x^2 - 1}$

infinite discontinuities at $x = \pm 1$
since $\lim_{x \to 1^-} f(x) = \lim_{x \to -1^-} f(x) = -\infty$

iii. $f(x) = \frac{x^2 - 3}{|x^2 - 3|}$

jump discontinuity at $x = \pm\sqrt{3}$
since $\lim_{x \to \sqrt{3}^-} f(x) \neq \lim_{x \to \sqrt{3}^+} f(x)$
and $\lim_{x \to -\sqrt{3}^-} f(x) \neq \lim_{x \to -\sqrt{3}^+} f(x)$

iv. $f(x) = \frac{x - 4}{x^2 - 16}$

infinite discontinuity at $x = -4$
since $\lim_{x \to -4^+} f(x) = \infty$
removable discontinuity at $x = 4$

since $\lim_{x \to 4} f(x) \neq f(x)$

3. (10 points) Find a value for the constant *k*, if possible, that will make *f* continuous.

$$f(x) = \begin{cases} 7x - 2 & x \le 1 \\ kx^2 & x > 1 \end{cases}$$

ANS: By definition of continuity:
i. $f(1)$ is defined [$f(1) = 7(1) - 2 = 5$]
ii. $\lim_{x\to 1} f(x)$ must exist. We know $\lim_{x\to 1^+} f(x) = 5$, so $\lim_{x\to 1^-} f(x) = 5$. The latter means that $kx^2 = 5$ when x approaches 1. Thus $k = 5$.
iii. $\lim_{x\to 1} f(x) = f(1) = 5$

4. (20 points) Find the limits, <u>algebraically</u>, if they exist. If the limit does not exist, explain.

a. $\lim_{t\to 1} \dfrac{t^3 - t^2 + 2t - 2}{t^2 - 3t + 2}$ **ANS:** $\lim_{t\to 1} \dfrac{(t-1)(t^2+2)}{(t-2)(t-1)} = \lim_{t\to 1} \dfrac{(t^2+2)}{(t-2)} = \dfrac{1^2+2}{1-2} = -3$

b. $\lim_{x\to 4} \dfrac{3-x}{x^2 - 2x - 8}$ **ANS:** $\lim_{x\to 4} \dfrac{3-x}{(x-4)(x+2)}$ = DNE since 0 in denom.

c. $\lim_{x\to 1^+} \dfrac{x^4 - 1}{x - 1}$ **ANS:** $\lim_{t\to 1} \dfrac{(x^2+1)(x-1)(x+1)}{(x-1)} = \lim_{t\to 1}(x^2+1)(x+1) = (1^2+1)(1+1) = 4$

d. $\lim_{x\to 5} \sqrt{x^3 - 3x - 1}$ **ANS:** $\lim_{x\to 5} \sqrt{(5)^3 - 3(5) - 1} = \sqrt{109}$

e. $\lim_{y\to 9} \dfrac{4-y}{2-\sqrt{y}}$ **ANS:** $\lim_{y\to 9} \dfrac{4-y}{2-\sqrt{y}} \dfrac{2+\sqrt{y}}{2+\sqrt{y}} = \lim_{y\to 9} \dfrac{(4-y)\left(2+\sqrt{y}\right)}{4-y} = \lim_{y\to 9}\left(2+\sqrt{y}\right) = 2 + \sqrt{9} = 5$

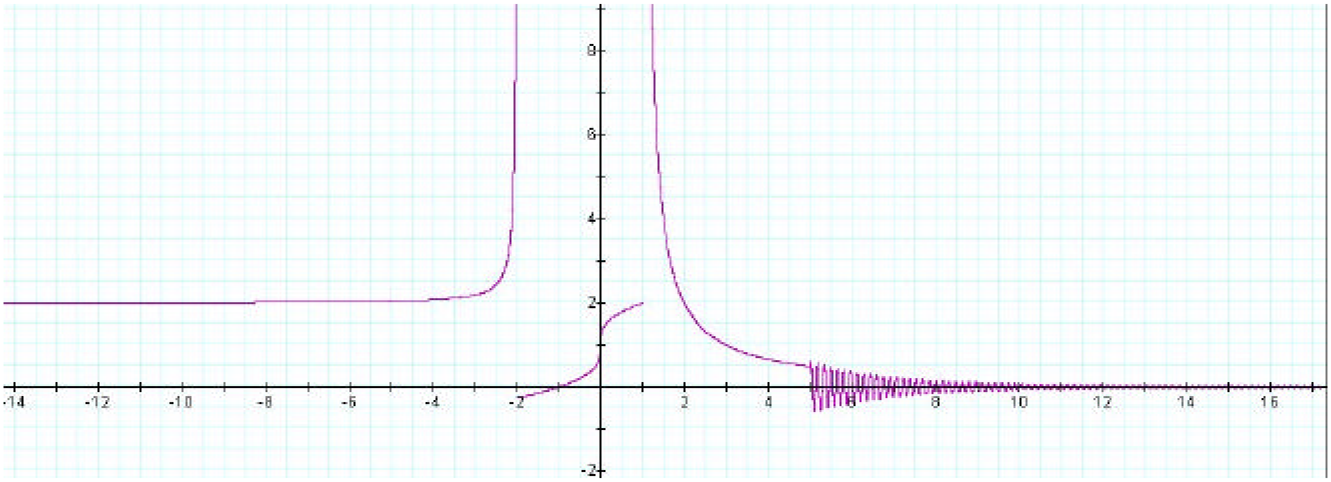
f. $\lim_{x\to 0} \dfrac{\sqrt{x+1} - 1}{x}$ **ANS:** $\lim_{x\to 0} \dfrac{\sqrt{x+1} - 1}{x} \dfrac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \lim_{x\to 0} \dfrac{x+1-1}{x(\sqrt{x+1}+1)} =$

$\lim_{x\to 0} \dfrac{1}{(\sqrt{x+1}+1)} = \dfrac{1}{\sqrt{0+1}+1} = \dfrac{1}{2}$

5. (10 points) Use the Intermediate Value Theorem to show that the polynomial function $f(x) = x^3 + 2x - 1$ has a root in the interval (0, 1). Be specific on the use of the theorem.

ANS: First, need to show that $f(x)$ is continuous on the interval [0, 1]. Since $f(x)$ is a polynomial function, $f(x)$ is continuous on [0, 1]. Next, we need to find $f(0) = -1$ and $f(1) = 2$. Since $-1 < 0 < 2$, there is a number c in (0, 1) such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $0 = x^3 + 2x - 1$ in the interval (0, 1).

6. (10 points) Find the equations of the asymptotes of the function graphed below. Explain the answer in terms of limits.



ANS:

Horizontal asymptotes are found by evaluating the limits at infinity:

$\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 2$, so $y = 0$ and $y = 2$ are horizontal asymptotes.

Vertical asymptotes are found by finding infinite limits:

$\lim_{x\to-2} f(x) = \infty$ and $\lim_{x\to1} f(x) = \infty$, so $x = -2$ and $x = 1$ are vertical asymptotes.

7. (10 points) Let

$$h(x) = \begin{cases} 2x - x^2 & 0 \le x \le 2 \\ 2 - x & 2 < x \le 3 \\ x - 4 & 3 < x < 4 \\ p & x \ge 4 \end{cases}$$

For each of the following numbers 2, 3 and 4, determine whether *h* is continuous at the number, continuous from the right or continuous from the left. Explain.

ANS: By definition of continuity:

i. $f(2) = 2(2) - (2)^2 = 0$

$f(3) = 2 - 3 = -1$

$f(4) = \pi$

So, $f(2)$, $f(3)$ and $f(4)$ are defined.

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ii. Consider $x = 2$:

$\lim_{x \to 2^-} h(x) = 2(2) - (2)^2 = 0$ and $\lim_{x \to 2^+} h(x) = 2 - 2 = 0$ so, $\lim_{x \to 2} h(x)$ exists.

Consider $x = 3$:

$\lim_{x \to 3^-} h(x) = 2 - 3 = -1$ and $\lim_{x \to 3^+} h(x) = 3 - 4 = -1$ so, $\lim_{x \to 3} h(x)$ exists.

Consider $x = 4$:

$\lim_{x \to 4^-} h(x) = 4 - 4 = 0$ and $\lim_{x \to 4^+} h(x) = p$ so, $\lim_{x \to 4} h(x)$ does not exist. Thus, $h(x)$ is not continuous at $x = 4$.

iii. Consider $x = 2$:

$\lim_{x \to 2} h(x) = h(2)$. Thus, $h(x)$ is continuous at $x = 2$.

Consider $x = 2$:

$\lim_{x \to 3} h(x) = h(3)$. Thus, $h(x)$ is continuous at $x = 3$.

Consider $x = 4$:

$\lim_{x \to 4^+} h(x) = h(4)$. Thus, $h(x)$ is not continuous from the right at $x = 4$.

8. (10 points) The displacement (in feet) of a certain particle moving in a straight line is given by $s = 2t^3 - 5t$, where t is measured in seconds.

a. Find the average velocity from $t = 2$ to $t = 5$.

ANS: Average velocity is the slope of the secant line through the points (2, 6) and (5, 225):

$$m = \frac{225 - 6}{5 - 2} = 73$$

b. Estimate the instantaneous velocity at $t = 2$.

ANS: Instantaneous velocity is the slope of the tangent line through the point (2, 6). This can be estimated by the slope of the secant line that passes through (2, 6) and (2.00001, 6.00019):

$$m = \frac{6.00019 - 6}{2.00001 - 2} = 19.0001 = 19.$$