MAT 195 – Fall Quarter 2002
TEST 3 - Answers

NAME____

Show work and write clearly.

For #1-12, find the derivative. Simplify your answer.

1. (7 pts.) $y = \frac{1}{2}\left(x^4 + 7\right)$

ANS: $y = \frac{x^4}{2} + \frac{7}{2}$. Use power rule. So, $y' = 4\frac{x^3}{2} + 0 = 2x^3$

2. (7 pts.) $y = \frac{x^2e^x + 1}{5}$

ANS: $y = \frac{x^2e^x}{5} + \frac{1}{5}$. Use product rule for first term. $y' = \frac{2xe^x + x^2e^x}{5} + 0 = \frac{xe^x(2 + x)}{5}$

3. (7 pts.) $y = \frac{7 - 5\sqrt{x}}{x^6}$

ANS: Since denominator is a single factor, separate the numerator: $y = \frac{7}{x^6} - \frac{5\sqrt{x}}{x^6}$. Move x^6 to the numerator, rewrite sqrt(x) as $x^{1/2}$ and combine exponents in second term. So, $y = 7x^{-6} - 5x^{-11/2}$. Now use power rule. $y' = -42x^{-7} + \frac{55}{2}x^{-13/2}$.

4. (7 pts.) $y = \sqrt[3]{x} - \frac{1}{x}$

ANS: Rewrite: $y = x^{1/3} - x^{-1}$. Use power rule: $y' = \frac{1}{3}x^{-2/3} - (-1)x^{-2} = \frac{1}{3}x^{-2/3} + x^{-2}$

5. (7 pts.) $y = \left(x^3 + 7x^2 - 8\right)\left(2x^{-3} + x^{-4}\right)$

ANS: Use product rule: $y' = \left(3x^2 + 14x\right)\left(2x^{-3} + x^{-4}\right) + \left(x^3 + 7x^2 - 8\right)\left(-6x^{-4} - 4x^{-5}\right)$

6. (7 pts.) $y = \frac{3x^2}{5x - 3}$

ANS: Use quotient rule: $y' = \frac{(5x - 3)6x - 3x^2(5)}{(5x - 3)^2} = \frac{30x^2 - 18x - 15x^2}{(5x - 3)^2} = \frac{15x^2 - 18x}{(5x - 3)^2}$

7. (7 pts.) $y = \left(2x^7 - x^2\right)\left(\frac{x - 5e^x}{x + 1}\right)$

ANS: You will need both product and quotient rules. By product rule:

$$y' = \frac{d}{dx}\left(2x^7 - x^2\right)\left(\frac{x - 5e^x}{x + 1}\right) + \left(2x^7 - x^2\right)\frac{d}{dx}\left(\frac{x - 5e^x}{x + 1}\right).$$

Use quotient rule to find

$$\frac{d}{dx}\left(\frac{x - 5e^x}{x + 1}\right) = \frac{(x + 1)\left(1 - 5e^x\right) - (1)\left(x - 5e^x\right)}{(x + 1)^2} = \frac{(x + 1)\left(1 - 5e^x\right) - (1)\left(x - 5e^x\right)}{(x + 1)^2} = \frac{1 - 5xe^x}{(x + 1)^2}.$$

So, $y' = \left(14x^6 - 2x\right)\left(\frac{x - 5e^x}{x + 1}\right) + \left(2x^7 - x^2\right)\frac{1 - 5xe^x}{(x + 1)^2}$

8. (7 pts.) $y = 2\cos x - 3\sin x$

ANS: $y' = 2(-\sin x) - 3\cos x = -2\sin x - 3\cos x$

9. (7 pts.) $y = \dfrac{\csc x}{\tan x}$

ANS: Use quotient rule:

$$y' = \frac{(\tan x)\,{}^{d}\!/_{dx}(\csc x) - {}^{d}\!/_{dx}(\tan x)(\csc x)}{\tan^2 x} = \frac{(\tan x)(-\csc x\cot x) - (\sec^2 x)(\csc x)}{\tan^2 x}.$$

This simplifies to: $y' = \dfrac{-\csc x - (\sec^2 x)(\csc x)}{\tan^2 x}$ …This simplifies further. Various answers for simplification were accepted.

10. (7 pts.) $y = x^3 \tan x - x\cos x$

ANS: Use product rule for each term.

$$y' = {}^{d}\!/_{dx}(x^3)(\tan x) + (x^3)\,{}^{d}\!/_{dx}(\tan x) - \left({}^{d}\!/_{dx}(x)(\cos x) + (x)\,{}^{d}\!/_{dx}(\cos x)\right)$$

$$= 3x^2(\tan x) + (x^3)\sec^2 x - (1(\cos x) + (x)(-\sin x)) = 3x^2\tan x + x^3\sec^2 x - \cos x + x\sin x$$

11. (7 pts.) $y = \dfrac{\sin x\sec x}{1 + x\tan x}$

ANS: You can simplify y to: $y = \dfrac{\tan x}{1 + x\tan x}$. Use quotient rule:

$$y' = \frac{(1 + x\tan x)\,{}^{d}\!/_{dx}\tan x - {}^{d}\!/_{dx}(1 + x\tan x)\tan x}{(1 + x\tan x)^2}$$

To find ${}^{d}\!/_{dx}(1 + x\tan x)$ use product rule:

$${}^{d}\!/_{dx}(1) + {}^{d}\!/_{dx}(x\tan x) = 0 + (1)\tan x + x\sec^2 x$$

So, $y' = \dfrac{(1 + x\tan x)\sec^2 x - (\tan x + x\sec^2 x)\tan x}{(1 + x\tan x)^2} = \dfrac{\sec^2 x - \tan^2 x}{(1 + x\tan x)^2} = \dfrac{1}{(1 + x\tan x)^2}$

12. (7 pts.) $y = \sin^2\left(\dfrac{1}{x}\right)$

ANS: Use chain rule:

Let $f(x) = x^2$, $g(x) = \sin x$ and $h(x) = \dfrac{1}{x}$. So, $y = f(g(h(x)))$.

By the chain rule, $y' = f'(g(h(x)))\cdot g'(h(x))\cdot h'(x) = 2\sin\left(\dfrac{1}{x}\right)\cdot\cos\left(\dfrac{1}{x}\right)\cdot\left(-x^{-2}\right)$.

13. (5 pts.) At what point(s) does the graph of the equation $y = x^4 - x^3 + x^2$ have a horizontal tangent line?

ANS: To find the slope of the tangent line, find the derivative: $y' = 4x^3 - 3x^2 + 2x^1$. When the tangent line is horizontal, the slope is 0. Set $y' = x^3 - 3x^2 + 2x^1 = 0$ and solve for *x*. So, $y' = x(4x^2 - 3x + 2) = 0$. Use quadratic formula to find the two solutions: $x = \dfrac{3 \pm \sqrt{41}}{8}$. The remaining horizontal tangent lines occurs at *x* = 0.

14. (6 pts.) a. Use the definition of the derivative at a point to find the derivative $f'(a)$ of $f(x) = x^3 + 5$ at $a = -2$.

b. Find the equation of the tangent line to $f(x)$ at $x = -2$.

ANS:

a. $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h} = \lim\limits_{h \to 0} \dfrac{(a+h)^3 + 5 - \left(a^3 + 5\right)}{h}$

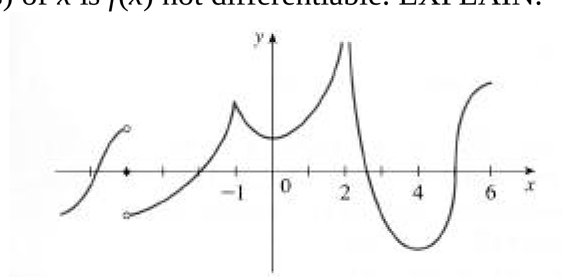
$= \lim\limits_{h \to 0} \dfrac{(a^3 + 3a^2h + 3ah^2 + h^3) + 5 - \left(a^3 + 5\right)}{h} = \lim\limits_{h \to 0} \dfrac{3a^2h + 3ah^2 + h^3}{h}$

$= \lim\limits_{h \to 0} \dfrac{h\left(3a^2 + 3ah^1 + h^2\right)}{h} = \lim\limits_{h \to 0}\left(3a^2 + 3ah^1 + h^2\right) = 3a^2$.

So, $f'(-2) = 3(-2)^2 = 12$ is the slope of the tangent line at $x = -2$.

b. $f(-2) = (-2)^3 + 5 = -3$. So, the equation of the tangent line is: $y + 3 = 12(x + 2)$

15. (5 pts.) For what value(s) of *x* is *f*(*x*) not differentiable. EXPLAIN.



ANS:

$x = -4$ because *f*(*x*) is discontinuous (jump).

$x = -1$ because *f*(*x*) has a cusp.

$x = 2$ because *f*(*x*) is discontinuous (infinite).

$x = 5$ because *f*(*x*) has vertical tangent.