MAT 195 – Spring Quarter 2002
TEST 4 – Answers

NAME____

Show work and write clearly.

1. Find an antiderivative for the following functions:

a. $f(t) = 2t^2 + 3t^3 + 4t^4$ **ANS:** $F(t) = \frac{2}{3}t^3 + \frac{3}{4}t^4 + \frac{4}{5}t^5 + C$

b. $f(x) = x^3 + 5\sqrt{x} - \frac{2}{x^2}$ **ANS:** $F(x) = \frac{1}{4}x^4 + 5 \cdot \frac{2}{3}t^{3/2} + 2x^{-1} + C$

c. $f(x) = x^{3/2} + \frac{\sqrt{x}}{5} - \frac{2}{x}$ **ANS:** $F(x) = \frac{2}{5}x^{5/2} + \frac{2}{3} \cdot \frac{x^{3/2}}{5} - 2\ln|x| + C$

2. The temperature change, T, in a patient generated by a dose, D, of a drug is given by $T(D) = \left(\frac{C}{2} - \frac{D}{3}\right)D^2$ where C is a positive constant. What dosage maximizes the temperature change? Explain.

ANS: The critical value(s) are where the derivative is zero or does not exist. $T'(D) = CD - D^2 \overset{set}{=} 0$. This implies $D = 0$ or $D = C$. Use the second derivative test to check if these are max or min values. The second derivative is: $T''(D) = C - 2D$. So, $T''(0) = C$ which is > 0 and $T''(C) = C - 2C$ which is < 0. Thus, the dosage which maximizes the temperature change is C.

3. The graph of g is shown below. The results from the left, right, midpoint and trapezoid rules used to approximate $\int_0^1 g(t)dt$, with the same number of subdivisions for each rule, are as follows: 0.601, 0.632, 0.633, 0.664.

a. Match each rule with its approximation. Explain.

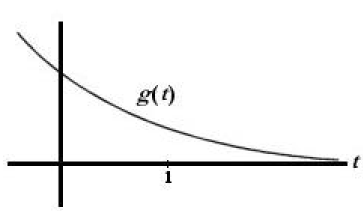
ANS:

RHS	MID	TRAP	LHS
0.601	0.632	0.633	0.664

The LHS and RHS are the worst estimates of the definite integral. The LHS is an overestimate for a decreasing function and the RHS is an overestimate. The midpoint sum is an underestimate for a concave up function and the trapezoid sum is an overestimate for a concave up function.

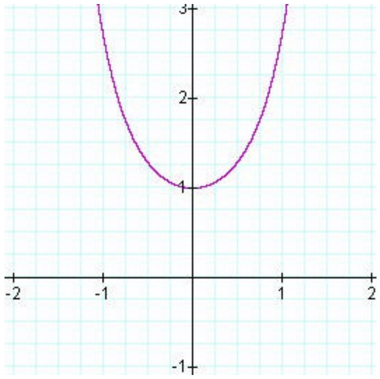
b. Between which two approximations does the true value of the integral lie? Explain.

ANS: The true value of the definite integral lies between 0.632 and 0.633 because the midpoint sum is an underestimate for a concave up function and the trapezoid sum is an overestimate for a concave up function.



4. Is $\int_{-1}^{1} e^{x^2}\,dx$ positive, negative or zero? Explain.

ANS: The graph of $y = e^{x^2}$ looks like:



The area between the curve and the *x*-axis is above the *x*-axis, so the definite integral is positive.

5. Given numbers a_1, a_2, a_3, let $f(x) = (x - a_1)^2 + (x - a_2)^2 + (x - a_3)^2$. Where is $f(x)$ a minimum? Explain.
ANS: Find critical value(s) by setting the derivative to zero. The easiest way to find the derivative is to expand each parentheses: $f(x) = x^2 - 2a_1x + a_1{}^2 + x^2 - 2a_2x + a_2{}^2 + x^2 - 2a_3x + a_3{}^2$ which simplifies to: $f(x) = 3x^2 - 2(a_1 + a_2 + a_3)x + a_1{}^2 + a_2{}^2 + a_3{}^2$. The derivative is: $f'(x) = 6x - 2(a_1 + a_2 + a_3) \overset{set}{=} 0$. The critical value is: $x = \dfrac{a_1 + a_2 + a_3}{3}$. The second derivative test may tell us if the function is a minimum at this critical value. $f''(x) = 6$ and because the second derivative is greater than zero, the function has a minimum at $x = \dfrac{a_1 + a_2 + a_3}{3}$.

6. Draw up a table of left- and right-hand sums with 2, 10, 50 and 250 subdivisions. Observe the limit to which your sums are tending as the number of subdivisions gets larger, and estimate the value of the definite integral.

$$\int_{0.2}^{3} \sin\left(\frac{1}{x}\right)dx$$

ANS:

n	LHS	RHS
2	-0.5234	1.2772
10	1.3116	1.6717
50	1.4980	1.5700
250	1.5253	1.5340

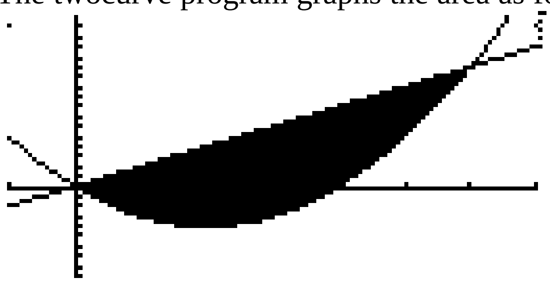
There are various possible estimates for the definite integral. One possibility is 1.5297 – the average of the LHS and RHS.

7. Find the intersection points of the following curves. Estimate the area enclosed by the given curves using 10 subdivisions and the midpoint rule. Check your answer with the twocurve calculator program. Sketch the region. $f(x) = 2x$ and $g(x) = x^2 - 4x$

ANS: To find the intersection points, $f(x) \overset{set}{=} g(x)$ and solve for x. That is, $2x \overset{set}{=} x^2 - 4x$ implies $x^2 - 6x = 0$ which implies x = 0, 6. Using the allsums program on your calculator, estimate the definite integrals: $\int_0^6 2x\,dx$ and $\int_0^6 \left(x^2 - 4x\right)dx$ by finding the midpoint sum with 10 divisions. These estimates are:

$\int_0^6 2x\,dx = 36$ and $\int_0^6 \left(x^2 - 4x\right)dx = -0.18$. The area between the two curves is $\int_0^6 2x\,dx - \int_0^6 \left(x^2 - 4x\right)dx$ (i.e., the definite integral of the 'top' function subtracted by the definite integral of the 'bottom' function), which is 36 – (–0.18) = 36.18.

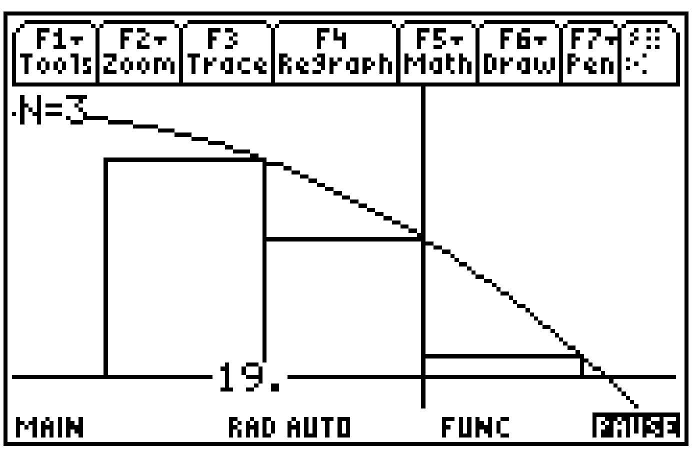
The twocurve program graphs the area as follows:



The area according to this program is 36. Our answer from above is an estimate since we used the midpoint sums.

8. (a) Estimate the area under the graph of $f(x) = -x^2 - 5x + 7$ from x = –2 to x = 1 using three subdivisions and right endpoints. Sketch the curve and the approximating rectangles. Is the estimate an underestimate or an overestimate? Explain.

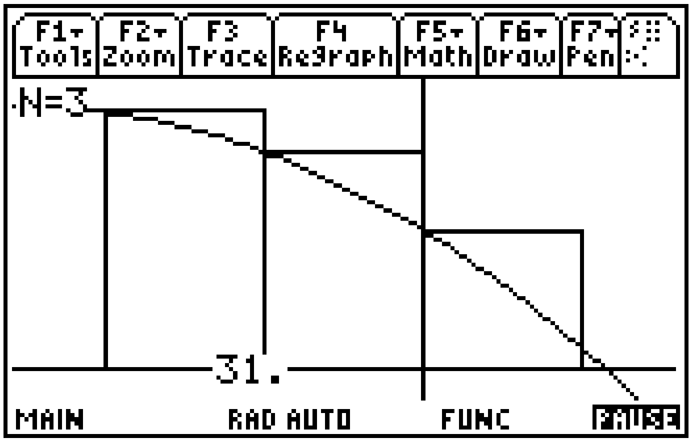
ANS:



This is an underestimate since the function is decreasing.

(b) Repeat using left endpoints.

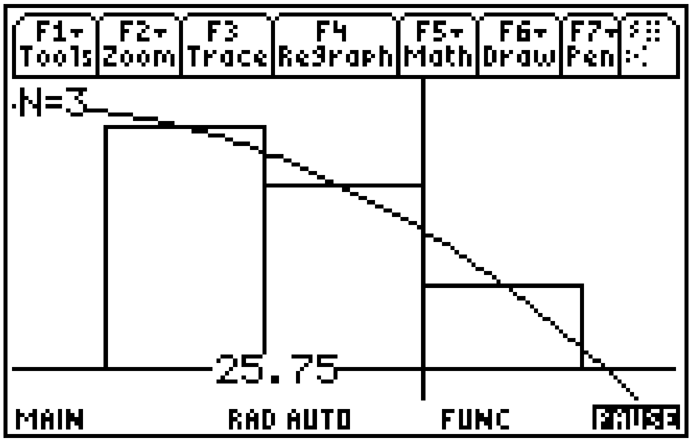
ANS:



This is an overestimate since the function is decreasing.

(c) Repeat using midpoints.

ANS:



This is an overestimate since the function is concave down.

(d) Which appears to be the best estimate? Explain.

ANS: The midpoint sum is the best estimate because each approximating rectangle has an area missing underneath the curve and an almost equal area above the curve.