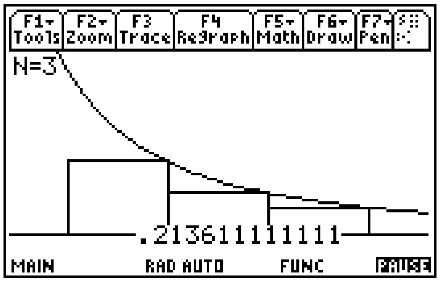
MAT 254 – Fall Quarter 2002
Test 1 - Answers

NAME____
Show work and write clearly.

1. (20 pts.) Without using the *allsums* program,

(a). Estimate the area under the graph of $f(x) = \frac{1}{x^2}$ from $x = 2$ to $x = 5$ using three approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate? Explain.

ANS:



$\Delta x = \frac{b-a}{n} = \frac{5-2}{3} = 1$ is the width of the approximating rectangles.

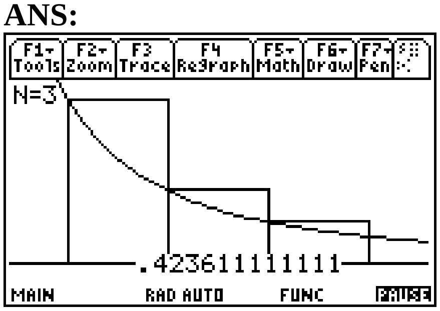
$RHS = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$.

$RHS = f(3)\Delta x + f(4)\Delta x + f(5)\Delta x = \frac{1}{3^2}(1) + \frac{1}{4^2}(1) + \frac{1}{5^2}(1) = \frac{769}{3600}$.

Since the function is decreasing on [2, 5], the RHS is an underestimate.

(b). Repeat using left endpoints.

ANS:



$\Delta x = \frac{b-a}{n} = \frac{5-2}{3} = 1$ is the width of the approximating rectangles.

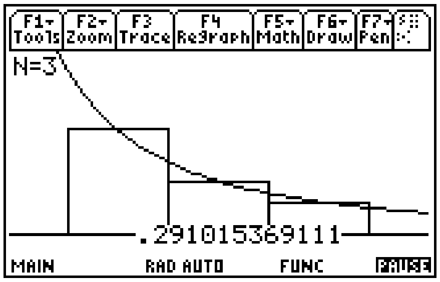
$LHS = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x$.

$LHS = f(2)\Delta x + f(3)\Delta x + f(4)\Delta x = \frac{1}{2^2}(1) + \frac{1}{3^2}(1) + \frac{1}{4^2}(1) = \frac{61}{144}$.

Since the function is decreasing on [2, 5], the LHS is an overestimate.

(c). Repeat using midpoints.

ANS:



$\Delta x_i = \dfrac{b-a}{n} = \dfrac{5-2}{3} = 1$ is the width of the approximating rectangles.

$$MIDPT = f\left(\frac{x_0 + x_1}{2}\right)\Delta x + f\left(\frac{x_1 + x_2}{2}\right)\Delta x + f\left(\frac{x_2 + x_3}{2}\right)\Delta x.$$

$$MIDPT = f(2.5)\Delta x + f(3.5)\Delta x + f(4.5)\Delta x = \frac{1}{2.5^2}(1) + \frac{1}{3.5^2}(1) + \frac{1}{4.5^2}(1) \cong 0.2910$$

Since the function is concave up on [2, 5], the MIDPT is an underestimate.

(d). Which gives the best estimation? Explain.

ANS: The MIDPT sum is the best estimate because each approximating rectangle has an overestimate and underestimate component.

2. (50 pts.) Using Part 2 of the Fundamental Theorem of Calculus to evaluate the integral, or explain why it does not exist. For trig functions, you may estimate the answer to 4 decimal places.

a. $\displaystyle\int_1^{\sqrt{3}} \frac{6}{1+x^2}\,dx$ **ANS:** $\displaystyle 6\tan^{-1} x\Big|_{x=1}^{\sqrt{3}} = 6\left(\tan^{-1}\sqrt{3} - \tan^{-1} 1\right) = \frac{\pi}{2}$

b. $\displaystyle\int_1^{9} \frac{5}{3x}\,dx$ **ANS:** $\displaystyle \frac{5}{3}\int_1^9 x^{-1}dx = \frac{5}{3}(\ln x)\Big|_{x=1}^{9} = \frac{5}{3}(\ln 9 - \ln 1) = \frac{5}{3}\ln 9 \approx 3.6620$

c. $\displaystyle\int_{-1}^{-3} \frac{2}{x^6}\,dx$ **ANS:** $\displaystyle 2\int_{-1}^{-3} x^{-6}dx = 2\left(\frac{1}{-5}x^{-5}\right)\Bigg|_{x=-1}^{-3} = -\frac{2}{5}\left((-3)^{-5} - (-1)^{-5}\right) = -\frac{2}{5}\left(-\frac{1}{243} + 1\right)$

d. $\displaystyle\int_{-\pi/3}^{-\pi/2} \sec x \tan x\sqrt{1+\sec x}\,dx$ **ANS:** DNE because sec(–π/2) is undefined.

e. $\displaystyle\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$ **ANS:** Use substitution – let $u = \sin^{-1} x$, then $du = \dfrac{1}{\sqrt{1-x^2}}\,dx$. When $x = 0$, $u = 0$ and when $x = 1/2$, $u = \pi/6$. The integral is now $\displaystyle\int_0^{\pi/6} u\,du = \frac{1}{2}u^2\Big|_{u=0}^{\pi/6} = \frac{\pi^2}{72}$.

3. (10 pts.) Using Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

a. $g(u) = \int_u^3 \frac{1}{x + x^2}\,dx$ **ANS:** $-\int_3^u \frac{1}{x + x^2}\,dx \Rightarrow g'(u) = -\frac{1}{u + u^2}$

4. (10 pts.) Calculate the left-hand, right-hand, midpoint and trapezoid sums with 100 subdivisions. Which of these sums are overestimates and which are underestimates? Explain. Estimate the value of the definite integral. Explain. $\int_{-2}^{3} \left(1 + \sqrt{9 - x^2}\right)dx$

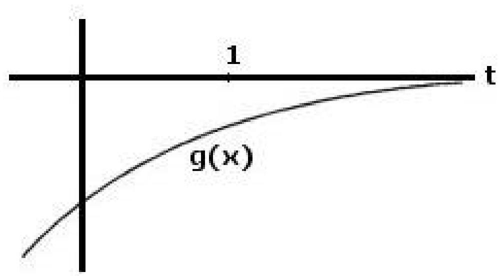
ANS:

Using allsums: LHS = 17.6385; RHS = 17.5266; MIDPT = 17.5902; TRAP = 17.5825.
The function is increasing on [-2, 0) and decreasing on (0, 3]. The function is an even function, i.e., it is symmetric w.r.t. the *y*-axis. Thus the area under the curve from $x = -2$ to $x = 0$ is the same as the area under the curve from $x = 0$ to $x = 2$. Thus, to determine the type of estimates for the RHS and LHS, we need only consider the area under the curve from $x = 2$ to $x = 3$. Since the function is decreasing on [2, 3], the RHS is an underestimate and the LHS is an overestimate. Since the curve is concave down on [–2, 3], TRAP is an underestimate and MIDPT is an overestimate. Finally, there are various answers for the estimate of the value of the definite integral – it must be between MIDPT and TRAP.

5. (10 pts.) The graph of *g* is shown below. The results from the left, right, midpoint and trapezoid rules used to approximate $\int_0^1 g(t)dt$, with the same number of subdivisions for each rule, are as follows: –0.601, –0.632, –0.633, –0.664.

a. Match each rule with its approximation. Explain.

b. Between which two approximations does the true value of the integral lie? Explain.



ANS: a. LHS = –0.664; RHS = –0.601; MIDPT = –0.632; TRAP = –0.633. The function is increasing on [0, 1], so the RHS is an overestimate and the LHS is an underestimate. Thus, the RHS needs to be the largest value and the LHS needs to be the smallest value (–0.601 > –0.664). The function is concave down, so the TRAP is an underestimate and the MIDPT is an overestimate.

b. The true value of the integral, A is between the MIDPT and TRAP sums. That is, TRAP < A < MIDPT.