Math 1431
Summer 2003 – Test #3 – Answers

NAME____
You are allowed to use your calculator. Show how you used the calculator to the questions below.
Explain all answers – answers with no explanation will receive only one–half credit.
Use complete sentences.

---

1. (20 points). Suppose you randomly answer a multiple choice test with 30 questions (each independent of each other). Suppose that each question has five possible answers only one of which is correct. Answer the following:
ANS: This is a binomial situation with $n = 30$ and $p = 1/5 = 0.2$.

a. Find the mean and standard deviation for the number of correct answers.
ANS: The mean is $m = np = 30(0.2) = 6$ and the standard deviation is $\sqrt{np(1-p)} = \sqrt{30(0.2)(0.8)} \cong 2.19$

b. What is the probability of answering more than 12 questions correctly?
ANS: Use 1–binomcdf(30, 0.2, 12) = 0.00311.

c. What is the probability of answering less than 15 questions correctly?
ANS: Use binomcdf(30, 0.2, 14) = 0.99977. We do not include 15 – we want less than 15 or less than or equal to 14!

d. What is the probability of answering exactly 15 questions correctly?
ANS: Use binompdf(30, 0.2, 15) = 0.000179.

---

2. (10 points). Seven chips marked 0, 1, 2, 3, 4, 5, 6 are placed in a box. Two chips are chosen randomly from the box. Let event A be the event that the first chip chosen is odd and event B be the event that the second chip is odd. The first chip is not replaced.

ANS: Here is the sample space:

~~0, 0~~	1, 0	2, 0	3, 0	4, 0	5, 0	6, 0
0, 1	~~1, 1~~	2, 1	3, 1	4, 1	5, 1	6, 1
0, 2	1, 2	~~2, 2~~	3, 2	4, 2	5, 2	6, 2
0, 3	1, 3	2, 3	~~3, 3~~	4, 3	5, 3	6, 3
0, 4	1, 4	2, 4	3, 4	~~4, 4~~	5, 4	6, 4
0, 5	1, 5	2, 5	3, 5	4, 5	~~5, 5~~	6, 5
0, 6	1, 6	2, 6	3, 6	4, 6	5, 6	~~6, 6~~

$n = 42$. Note that the number that is chosen with the first chip cannot be chosen with the second chip!

a. Find P(A), P(B) [extra credit: find P(A or B) and P(A and B)]
ANS:
P(A) = 18/42 = 3/7 (by counting the observations in the sample space).
P(B) = 18/42 = 3/7 (by counting).
P(A or B) = P(A) + P(B) – P(A and B) = 3/7 + 3/7 – 1/7 = 5/7.
P(A and B) = 6/42 = 1/7 (by counting).

b. Are A and B independent? Explain.
ANS: Since the first chip is NOT replaced, the two events are NOT independent.

---

3. (15 points). Seven chips marked 0, 1, 2, 3, 4, 5, 6 are placed in a box. Two chips are chosen randomly from the box. Let event A be the event that the first chip chosen is odd and event B be the event that the second chip is odd. The first chip is replaced.
ANS: See sample space for #2 above.

a. Find P(A), P(B), P(A or B) and P(A and B).
ANS:
P(A) = 3/7.
P(B) = 3/7.
P(A and B) = P(A)P(B) because the events are independent. So, P(A and B) = (3/7)(3/7) = 9/49.
P(A or B) = P(A) + P(B) – P(A and B) = 3/7 + 3/7 – 9/49 = 33/49.

b. Are A and B independent? Explain.
ANS: Since the first chip is replaced, the two events are independent.

---

4. (15 points). Two fair six-sided dice are tossed. Event A is the toss of a five on at least one die. Event B is sum of seven on the toss of both die. Find the following:
ANS: Here is the sample space:

1 + 1 = 2	2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	6 + 1 = 7
1 + 2 = 3	2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7	6 + 2 = 8
1 + 3 = 4	2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8	6 + 3 = 9
1 + 4 = 5	2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9	6 + 4 = 10
1 + 5 = 6	2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10	6 + 5 = 11
1 + 6 = 7	2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11	6 + 6 = 12

n = 36.

a. P(A), P(B), P(A|B), P(B|A).
ANS:
P(A) = P(5 on Die 1 or 5 on Die 2) = P(5 on Die 1) + P(5 on Die 2) – P(5 on both Die) =
6/36 + 6/36 – 1/36 = 11/36.
P(B) = P(sum of 7) = 6/36 = 1/6.
P(A|B) = P(5 on either Die given the sum is 7) = 2/6 = 1/3. (There are 6 cases in which the sum is seven, 2 of which have a 5 on one of the die.) Alternatively, P(A|B) = P(A and B)/P(B) = 2/36 ÷ 1/6 = 2/6 = 1/3.
P(B|A) = P(the sum is 7 given 5 on either Die) =2/11. (There are 11 cases in which a 5 appear on one of the die, 2 of which the sum is seven.) Alternatively, P(B|A) = P(A and B)/P(A) = 2/36 ÷ 11/36 = 2/11.

b. Are A and B independent?
ANS: No, because P(A|B)   P(A).

---

5. (30 points). The table below shows the preference of cola of different age groups:

	Under Age 15	**Ages 15-25**	**Ages 25-35**	**Total**
Cola 1	150	100	200	450
Cola 2	300	125	200	625
Cola 3	300	200	300	800
Total	750	425	700	1875

a. Find the probability that a randomly chosen person prefers Cola 1.
ANS: P(Cola 1) = 450/1875 = 6/25 = 0.24.

b. Find the probability that the age of a randomly chosen person is between 15 and 25.
ANS: P(Ages 15-25) = 425/1875 = 17/75 0.2267.

c. Find the probability that the age of a randomly chosen person is between 15 and 35.
ANS: P(Ages 15-35) = P(Ages 15-25 or Ages 25-35) = P(Ages 15-25) + P(Ages 25-35) = 425/1875 + 700/1875 = 1125/1875 = 3/5 = 0.6.

d. Find the probability that a randomly chosen person prefers Cola 3 given that the person is between 15 and 25 years old.
ANS: P(Cola 3 | Ages 15-25) = P(Cola 3 and Ages 15-25)/P(Ages 15-25) = 200/425 = 8/17 0.471.

e. Find the probability that a randomly chosen person is under 15 years old given that s/he prefers Cola 1.
ANS: P(Under Age 15 | Cola 1) = P(Cola 1 and Under Age 15)/P(Cola 1) = 150/450 = 1/3 0.333.

f. Is the age of individuals and the cola preference independent? Explain using the definition of independence.
ANS: NO, the age of individuals and the cola preference are not independent since P(Cola 3 | Ages 15-25) P(Cola 3). Note that P(Cola 3) = 800/1875 = 32/75.

---

6. (10 points). The mean and standard deviation of a random sample of 30 students' IQ at a certain college are 120.3 and 10.5 respectively. Find the 90%, 95% and 99% confidence intervals for the average IQ for all students in the school.
ANS: The 90% CI is 117.15 to 123.45. The 95% CI is 116.54 to 124.06. The 99% CI is 115.36 to 125.24.