

Math 1431
Spring 2003 – Test #3 – Answers

NAME _____

You are allowed to use your calculator. Show how you used the calculator to the questions below.

Explain all answers – answers with no explanation will receive only one-half credit.

Use complete sentences.

1. (20 points). Find the following probabilities:

a. A coin is tossed and a fair six-sided dice is rolled at the same time. What is the probability of tossing a head and rolling a 1 or 2?

ANS: These events are independent, so $P(H \text{ and } 1 \text{ or } 2) = P(H) \cdot P(1 \text{ or } 2) = (1/2)(1/3) = 1/6$.

b. Three green balls and one red ball are placed in a box. What is the probability of removing two green balls if each ball is replaced after it is removed?

ANS: These events are independent (because of the replacement), so $P(GG) = P(g) \cdot P(G) = (3/4)(3/4) = 9/16$.

c. Three green balls and three red balls are placed in a box. What is the probability of removing a second green ball if the first ball chosen was green and was not replaced after it is chosen?

ANS: These events are NOT independent, so $P(G) = 2/5$.

d. A student randomly guesses at 10 multiple-choice questions. Find the probability that the student guesses exactly 3 correctly. Each question has four possible answers with only one correct answer and each question is independent of every other question.

ANS: This is a binomial situation, where $n = 10$, $k = 3$, $p = 0.25$ (because there is a 1 in 4 chance of guessing an answer correctly). So, $P(X = 3) = \text{binompdf}(10, 0.25, 3) = 0.2503$.

e. From the information given in 1d, what is the probability of guessing more than 8 questions correctly?

ANS: This is a binomial situation, where $n = 10$, $k > 8$, $p = 0.25$. So, $P(X > 8) = 1 - P(X \leq 8) = 1 - \text{binomcdf}(10, 0.25, 8) = 0.00003$.

f. From the information given in 1d, let X be the number of questions guessed correctly. What is the mean of X ?

ANS: This is a binomial situation, where $n = 10$, $k > 8$, $p = 0.25$. So, the mean $= np = 10(0.25) = 2.5$.

2. (20 points). The mean of a random sample of 150 employees' salaries in some company is \$18,500 per year with a standard deviation of \$2500.

a. Find the 90% and 99% confidence intervals for the mean salary of all employees in the company.

ANS: Using ZInterval on TI83: the 90% CI for the mean salary of all employees in the company is \$18164 to \$18836 and the 99% CI for the mean salary of all employees in the company is \$17974 to \$19026.

b. Explain what the 90% confidence interval means.

ANS: 90% of CI from all samples of size 150 contain the population mean.

c. Find the minimum sample size needed for a margin of error of $\pm \$300$ and an 95% confidence interval.

ANS: The ZSAMPLE program on TI83 returns $n = 266.7$, so the minimum sample size needed is 267.

3. (20 points). Here are measurements (in millimeters) of a critical dimension on a sample of automobile engine crankshafts:

224.120	224.001	224.017	223.982	223.989	223.961	223.960	224.089
223.987	223.967	223.902	223.980	224.089	224.057	223.913	223.999

The manufacturing process is known to vary normally with standard deviation of 0.060 mm. The process mean is supposed to be 224 mm. Do these data give evidence that the process mean is not equal to the target value of 224 mm?

a. State H_0 and H_a .

ANS: $H_0: \mu = 224$
 $H_a: \mu \neq 224$

b. Carry out a significance test with a $\alpha = 0.05$ significance level to justify your.

ANS: Using ZTest on the TI83, the p -value is 0.9568.

c. State your conclusion.

ANS: Since the p -value $> \alpha$, we fail to reject H_0 . There is not enough information to accept H_0 that the process mean is equal to the target value of 224 mm.

4. (20 points). A candy company sells milk and dark chocolates in 1-pound and 2-pound boxes. Here are the numbers of boxes sold one month:

SALES	Milk Chocolate	Dark Chocolate	Total
1 pound boxes	350	300	650
2 pound boxes	200	150	350
Total	550	450	1000

a. Find the probability that a box of chocolate sold is dark chocolate.

ANS: $P(D) = 450/1000 = 9/20$.

b. Find the probability that a box contains milk chocolates given it is a 2-pound box.

ANS: $P(M | 2\#) = 200/350 = 4/7$.

c. Find the probability that a box is a 1-pound box given it contains dark chocolates.

ANS: $P(1\# | D) = 300/450 = 2/3$.

d. Are the type of chocolates and the size of the boxes independent? Explain using the definition of independence.

ANS: The type of chocolates and the size of the boxes are independent only if $P(M | 2\#) = P(M)$. $P(M | 2\#) = 4/7$ (from b. above) and $P(M) = 550/1000 = 11/20$. Since $P(M | 2\#) \neq P(M)$, then the type of chocolates and the size of the boxes are NOT independent.

5. (10 points). A merchant claims that the average age of customers who purchase a certain brand of jeans is 15. A sample of 35 customers had an average of 15.6 years with standard deviation of one year. Test the claim with an $\alpha = 0.05$ significance level.

ANS: $H_0: \mu = 15$
 $H_a: \mu \neq 15$

Using ZTest on the TI83, the p -value is 0.00039. Since the p -value $< \alpha$, we reject H_0 . That is, the average age of customers who purchase a certain brand of jeans is NOT 15.

Short Answers

6. (5 points). In a one-tailed statistical test of hypotheses, explain when we can reject H_0 .

ANS: We can reject H_0 in a one-tailed statistical test if p -value $\leq \alpha$.

7. (5 points). Provide a setting in which a random variable has a binomial distribution. Explain why this is a binomial setting.

ANS: Many possible settings are possible.

Extra Credit

(3 points). Write the formula for the z -statistic.

ANS: The z -statistic is: $z = \frac{\bar{x} - \mathbf{m}}{\mathbf{s} / \sqrt{n}}$.