Situation 35: Solving Quadratic Equations
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Prompt

In Algebra 1 classes some students solve quadratic equations as follows:

Solve for x: $x^2 = x + 6$.
Students' work:
$\sqrt{x^2} = \sqrt{x+6}$
$x = \sqrt{x+6}$

Commentary

Mathematical Foci

Mathematical Path 1

The solutions to the three equations can be compared graphically to determine whether the equations are equivalent. Equations are equivalent if they have the same solutions.

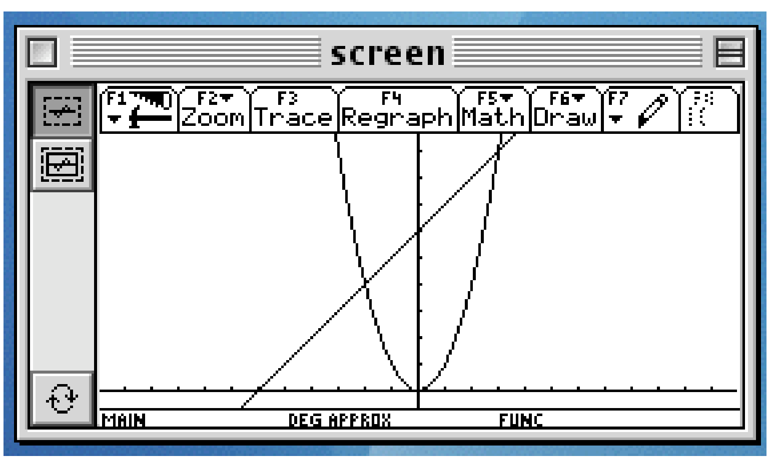

Figure 1. $Y1 = x^2$ and $Y2 = x + 6$. Y1 and Y2 intersect at x = -2, 3.

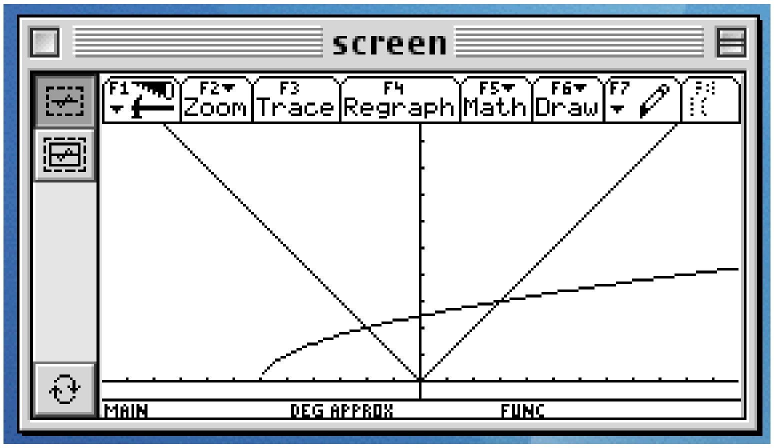

Figure 2. $Y3 = \sqrt{x^2}$ and $Y4 = \sqrt{x+6}$. Y3 and Y4 intersect at x = -2, 3.

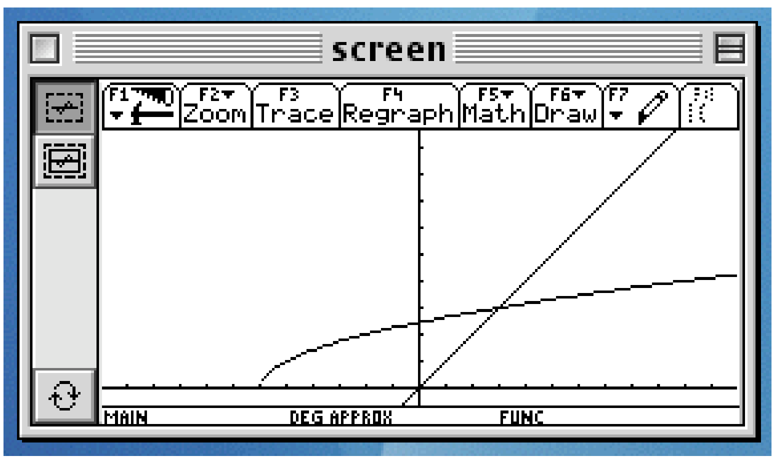

Figure 3. Y5 = x and $Y4 = \sqrt{x+6}$. Y4 and Y5 intersect at x = 3

The last equation, $x = \sqrt{x+6}$, is not equivalent to the other two equations since its solution is not the same as that of the other equations.

Mathematical Path 2

The graphs of $\sqrt{x^2} = \sqrt{x+6}$ can lead to a discussion of the equivalence of $f(x) = \sqrt{x^2}$ and $g(x) = |x|$.

The two functions have the same domain and give rise to the same set of points.

So, $\sqrt{x^2} = |x| = \begin{cases} x, & if\ x > 0 \\ 0, & if\ x = 0 \\ -x, & if\ x < 0 \end{cases}$.

Mathematical Path 3

The quadratic formula can be used to solve $x^2 = x + 6$.

$$x^2 = x + 6$$
$$x^2 - x - 6 = 0$$
$$x = \frac{1 \pm \sqrt{1 - 4(1)(-6)}}{2} = \frac{1 \pm 5}{2} = 3, -2$$

Mathematical Path 4

The quadratic equation, $x^2 = x + 6$, can be solved by factoring and applying the zero product property.

$$x^2 = x + 6$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x - 3 = 0,\ x + 2 = 0$$
$$x = 3, -2$$

References

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