Situation 3: Inverse Functions

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Main ideas include:

As we discussed conceptions of inverse are at stake in this problem. In this case, the undefined operation seems to be the difficulty. In a broad sense, an inverse is related to the concept of identity in a given context. So in the multiplicative context applying the inverse gives us the identity element, 1. In a functional context, the identity elements are taken to be the elements themselves (which is usually a set) so the inverse function maps all elements of the domain onto themselves. One difficulty or difference here might be why in one case we take the identity to be an entire set whereas in the other case we take the identity to be a singular element.

To me it seems like functional inverses come more from transformational geometry whereas multiplicative inverses come from number and operations concepts. I say this because the question arises in transformational geometry of how to undo a transformation that has transformed all (or some) of the points in the plane whereas a multiplicative inverse seems to come from thinking about how to undo making a quantity a certain amount larger.

Another main idea might be seen as the connection between the abstract algebra definitions given in mathematical path 2 and the geometric representation given in mathematical path 1. If you assume that $f(g(x)) = x$ then it is possible to show that $y = x$ is a line of reflection for the functions f,g. If you assume that $y = x$ is a line of reflection for the functions f,g, then it is possible to demonstrate that $f(g(x)) = x$. This exercise might provide some justification for why you might assume that the line $y = x$ is a line of reflection for the inverse.

Another large idea that may be implicit here is the concept of function. Often, it seems like students rely on their understanding of rational numbers and operations to interpret the concept of function (and hence it might make sense to interpret the concept of the inverse of a function as a multiplicative inverse). Application of this understanding works to some extent in polynomial contexts. However, application of this understanding does not work for the sine function whose domain is more explicitly real numbers (or at least some of them); that is we often teach students that $\pi/2$ maps to 1 but this relationship (even pointwise) cannot be derived from operations on rational numbers.